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On Environmental Taxation under Uncertainty*

by

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Abstract

This paper addresses optimal taxation, when the relationship between consumption and environmental damage is uncertain and treated as a random variable by policy makers. The main purpose is to analyze how additional uncertainty about this relationship affects the optimal unit tax on the consumption good that is causing environmental damage. We find that the optimal response to this tax depends on (i) the attitudes towards risk and (ii) how other policy instruments affect the demand for the good that is causing damage to the environment.

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1. Introduction

Much research effort has been put into studying 'green taxes' as a means of improving the resource allocation. It is now recognized that the proper design of environmental taxation does not only depend on the environmental damage caused by e.g. production or consumption; it also depends on other distortions in the economy[1]. However, as far as we know, there are very few attempts to incorporate uncertainty into the analysis of environmental taxation[2]. This is somewhat surprising, since the environmental effects of production and consumption are almost always uncertain. In the context of the relationship between consumption/production and pollution, there are several types of uncertainty that may be of interest to study. First, there can be uncertainty about the magnitude (or severity) of the environmental damage caused by certain pollutants. We might be able to perfectly observe the quantity of a potential pollutant while, at the same time, not knowing its exact impact on the environment. A frequent example is the effects of carbon dioxide emissions, where the major issue is whether or not global warming may have severe impacts on the climate and, therefore, on the living conditions of mankind. A second type of uncertainty occurs when the emissions themselves are not perfectly observed. For example, it would be very hard to observe the exact amount of nitrogen and other emissions from individual cars.

This paper analyzes optimal environmental taxation under the first type of uncertainty mentioned above, where the environmental consequences of economic behavior are not known with perfect certainty. To operationalize this idea, we will assume that the relationship between the aggregate consumption of a certain good - to be called 'dirty' good - and the resulting environmental damage is uncertain, meaning that this relationship will have the character of a random variable. By assuming that the probability distribution for this random variable is known, we can formulate the optimal tax problem in terms of maximization of expected utility. The main purpose is to analyze how a mean preserving increase in the spread of the random environmental damage affects the optimal choice of environmental taxation.

[1] See e.g. Bovenberg and de Mooij (1994, 1996) and Bovenberg and van der Ploeg (1994).
[2] Exceptions are Aronsson (1998), dealing with the role of environmental taxation under uncertain timing of the development of new abatement technologies, and Aronsson et al. (1998), who address external effects related to the likelihood of nuclear accidents.

When the environmental consequences of consumption are not known with perfect certainty, the attitudes towards risk become important in the context of environmental policy. If agents do not become less risk averse when the environmental damage increases, it is not difficult to imagine a precautionary motive for environmental taxation. This means that uncertainty about the relationship between consumption and environmental damage may work to increase taxes, which are motivated by concern for the environment. At the same time, it is important to recognize that the government does not only impose taxes in order to internalize externalities; it also uses taxation from various sources to finance public goods. As a consequence, the optimal level of environmental taxation is likely to depend on how the other tax instruments available affect the demand for goods that are causing environmental damage. Therefore, the attitudes towards risk are, in general, not sufficient for determining the policy implications of the type of uncertainty to be analyzed here.

To simplify the analysis as much as possible, we shall disregard distributional policy objectives as well as the inherent dynamic nature of many pollutants by using a static representative agent model[3]. Environmental taxation will be introduced in terms of a unit tax on the dirty good, the consumption of which is causing the environmental damage. In the benchmark version of the model examined in Section 2, the government is assumed to be able to raise lump-sum taxes, meaning that the tax instruments available are the tax on the dirty consumption good and the lump-sum tax. In Section 3, the lump-sum tax will be replaced by a labor income tax. Finally, Section 4 concludes the paper.

[3] The more recent research has extended the study of environmental taxation to address intertemporal choice and distributional objectives. For example, Bovenberg and de Mooij (1997) and Aronsson (1999) analyze welfare implications of green tax reforms in a dynamic general equilibrium framework, whereas Pirtillä and Tuomala (1997) study optimal enviromental taxation in an economy where the tax system is designed to fulfill both efficiency and distributional objectives.

2. A Benchmark Model

The benchmark setting means that the government can finance its consumption by lump-sum taxes. This assumption will be relaxed in the next section, where the lump-sum tax is replaced by a labor income tax. Since we disregard distributional issues, the population is normalized to one.

The budget constraint of the representative agent is written

$$c + qx + wL = wH - T \tag{1}$$

where c and x are consumption of 'clean' and 'dirty' private goods, respectively, and L is leisure. The price of the dirty good is defined as $q = p + t$, where p is the fixed producer price and t a unit tax. We shall refer to t as an environmental tax. The price of the clean good has been normalized to one. The right hand side of equation (1) measures full income, where w is the wage rate, H a time endowment and T a lump-sum tax.

To be able to derive clear-cut results relating to the consequences of uncertainty, it has not been uncommon to assume an additive utility function[4]. We shall follow this approach in part and write the direct utility function of the representative individual as follows:

$$U = u(c, x, L) + \varphi(g) + v(bD) \tag{2}$$

where g is public consumption and D the aggregate consumption of the dirty private good or 'externality base'. Since the population has been normalized to equal one, we have $D = x$. Note that the representative individual takes the externality base as exogenous when solving his/her optimization problem. The term bD is interpreted to measure the environmental damage caused by the consumption of dirty goods. The parameter b will be explained below. We assume that $u(\cdot)$ is increasing in each argument, twice continuously differentiable and strictly quasiconcave. Regarding the

[4] See e.g. Johansson and Löfgren (1985) and Koskela and Ollikainen (1997, 1998).

other parts of the utility function, we assume $\varphi_g(g) > 0$, $\varphi_{gg}(g) < 0$, $v_z(z) < 0$ and $v_{zz}(z) < 0$, where $z = bD$.

From equation (2) it follows that the representative individual behaves as if he/she is maximizing $u(c, x, L)$ subject to equation (1), which means that the private optimization gives

$$c = c(q, w, T) \tag{3a}$$

$$x = x(q, w, T) \tag{3b}$$

$$L = L(q, w, T) \tag{3c}$$

and we assume that c, x and L are all normal goods.

Even if the externality base, D, is perfectly observed, the relationship between this externality base and the environmental damage is uncertain to policy makers. According to equation (2), the environmental damage is proportional to the aggregate consumption of dirty goods, and b is the factor of proportionality. We shall define

$$b = \bar{b} + qe \tag{4}$$

with $E(e) = 0$, $s_e^2 = 1$ and $\Pr(e > -\bar{b}/q) = 1$. We can interpret $b(e)$ as a positive random variable with mean $\bar{b}$ and standard deviation q. A mean preserving increase in the spread of this random variable is, therefore, measured by an increase in the standard deviation, q.

By substituting equations (3a), (3b) and (3c) into equation (2), and then taking expectations, we obtain the expected indirect utility function

$$E[V] = \Omega(q, w, T) + \varphi(g) + E[v(b\,x(q, w, T))] \tag{5}$$

The government chooses t, T and g such as to maximize the expected indirect utility subject to the budget constraint

$$t\,x(q,w,T)+T=g \tag{6}$$

Since our main purpose is to analyze the optimal tax part of the government's problem, we substitute equation (6) into equation (5)

$$MaxE[V]=\underset{t,T}{Max}\,\Omega(q,w,T)+\varphi(tx(q,w,T)+T)+E[v(b\,x(q,w,T))]$$

and the first order conditions can be written as[5]

$$E[V_t]=\Omega_q+\varphi_g(x+tx_q)+x_q E[b\,v_z(bx)]=0 \tag{7a}$$

$$E[V_T]=\Omega_T+\varphi_g(tx_T+1)+x_T E[b\,v_z(bx)]=0 \tag{7b}$$

where $z=bx$ and subindices denote partial derivatives. We assume that the tax revenue is an increasing function of t and T, i.e. $x+tx_q>0$ and $1+tx_T>0$, at the equilibrium. Given equations (7a) and (7b), which implicitly define the optimal t and T, one can use equation (6) to solve for the optimal public expenditure. We shall also require that the second order sufficient conditions for maximization are fulfilled.

Let us begin by interpreting the first order conditions. By solving equation (7b) for j_g, substituting into equation (7a) and using Roy's identity, we can rewrite equation (7a) in terms of an analogue to a well known optimal tax rule;

$$t=\frac{E[b\,v_z(bx)]}{\Omega_T} \tag{8}$$

[5] The first order condition for public consumption is implicit in equations (7a) and (7b). This condition means that the marginal utility of the public good is equal to the marginal cost of public funds.

Equation (8) means that the optimal environmental tax equals the expected marginal externality in real terms, where $\Omega_T < 0$ is the negative of the marginal utility of income. This result is a consequence of the assumption that the government can use lump-sum taxes to finance public consumption. It means that it is optimal to fully internalize the expected external effect. Note finally that equation (8) only gives the optimal environmental tax on an implicit form; both x and Ω_T depend on t.

Let us now turn to the main issue of this section; how a mean preserving increase in the spread of the (random) environmental damage affects the optimal environmental tax. Differentiating equations (7a) and (7b) with respect to t, T and q, we find

$$\begin{bmatrix} E[V_{tt}] & E[V_{tT}] \\ E[V_{Tt}] & E[V_{TT}] \end{bmatrix} \begin{bmatrix} \partial t / \partial q \\ \partial T / \partial q \end{bmatrix} = \begin{bmatrix} -x_q \{\partial E[b v_z(bx)] / \partial q\} \\ -x_T \{\partial E[b v_z(bx)] / \partial q\} \end{bmatrix} \tag{9}$$

and we can solve for $\partial t / \partial q$ and $\partial T / \partial q$ as follows;

$$\frac{\partial t}{\partial q} = \frac{[x_T E[V_{tT}] - x_q E[V_{TT}]]\{\partial E[b v_z(bx)] / \partial q\}}{A} \tag{10a}$$

$$\frac{\partial T}{\partial q} = \frac{[x_q E[V_{Tt}] - x_T E[V_{tt}]]\{\partial E[b v_z(bx)] / \partial q\}}{A} \tag{10b}$$

where A is the determinant corresponding to the Hessian matrix of equation system (9). Note that $E[V_{tt}] < 0$, $E[V_{TT}] < 0$ and $A = E[V_{tt}]E[V_{TT}] - \{E[V_{tT}]\}^2 > 0$ from the second order sufficient conditions for maximization.

To be able to analyze the implications of uncertainty for the optimal environmental tax, one has to specify the attitudes towards risk in greater detail. Since the worst case scenarios for some pollutants are ‘doomsday-like’, it appears to be reasonable to assume that the consumer will at least not become less risk averse when the environmental damage (or externality base) increases. Formally, the preferences are characterized by nondecreasing absolute risk aversion if

$$\frac{v_{zz}(z^1)}{v_z(z^1)} \geq \frac{v_{zz}(z^0)}{v_z(z^0)} \text{ for } z^1 > z^0$$

where $v_{zz} / v_z > 0$, since $v_z < 0$ and $v_{zz} < 0$. Consider the following result;

Lemma 1: *With nondecreasing absolute risk aversion,* $\partial E[b\, v_z(bx)] / \partial q < 0$.

Proof: see the Appendix.

Lemma 1 is a consequence of the assumption that the 'externality part' of the utility function is additive, implying that the impact of the variance parameter q on the marginal utility of the externality base is attributable to risk version. This is so because the consumption of dirty goods does not depend explicitly on q. We can now interpret equation (10a) as follows;

Proposition 1: *With nondecreasing absolute risk aversion, an increase in* q *will increase (decrease) the optimal environmental tax if, and only if,* $x_T E[V_{tT}] - x_q E[V_{TT}]$ *is negative (positive).*

To fully understand this result, note that since $x_q < 0$, then $x_q E[V_{TT}] > 0$. This term reflects the precautionary motive for environmental taxation mentioned in the introduction. Given the assumption about nondecreasing absolute risk aversion, this effect will work to increase the environmental tax as a response to additional uncertainty. The intuition is that one can counteract the disutility from additional uncertainty by reducing the consumption of dirty goods.

However, the consumption of dirty goods will also be reduced by higher lump-sum taxation, and even if $x_T < 0$ the term $x_T E[V_{tT}]$ can, in general, go in either direction. Therefore, without further assumptions, we cannot rule out the situation where the government responds to additional uncertainty by increasing the lump-sum tax and reducing the environmental tax. To rule out this possibility, one would have to impose restrictions on the magnitude of the 'income effects'; if x_T is sufficiently small, the optimal response to additional uncertainty will be to increase the environmental tax. By taking this argument to its extreme, we can derive;

Proposition 2: *With nondecreasing absolute risk aversion, and if the utility function is quasi-linear in the sense* $u(c,x,L)=c+f(x,L)$, *then* $\partial t/\partial q>0$, $\partial T/\partial q<0$ *and* $\partial g/\partial q=0$.

Proof: With the assumption of quasi-linearity, we can rewrite equations (10a) and (10b) to read

$$\frac{\partial t}{\partial q}=\frac{-x_q\{\partial E[v_z(bx)]/\partial q\}j_{gg}}{A}>0$$

$$\frac{\partial T}{\partial q}=\frac{x_q[x+tx_q]\{\partial E[v_z(bx)]/\partial q\}j_{gg}}{A}<0$$

Therefore, by using the budget constraint it follows that $\partial g/\partial q=0$. ■

The intuition behind Proposition 2 is as follows. Since quasi-linearity implies that the marginal utility of public consumption is equal to one at the optimum, and by the assumption that public consumption enters the utility function additively, it follows that public consumption will not be affected by additional uncertainty. In addition, the only policy instrument that can be used to reduce the consumption of dirty goods is the environmental tax. Therefore, the optimal response will be to increase the environmental tax and reduce the lump-sum tax so as to maintain the public sector equilibrium at the initial level of public expenditures.

3. Distortionary Labor Income Taxation

The budget constraint of the representative consumer is now given by

$$c+qx+wL=wH \qquad (11)$$

where $w = w(1-t)$ is the net wage rate and t the labor income tax rate. The outcome of private optimization changes to read (if we suppress the time endowment for notational convenience)

$$c = c(q,w) \tag{12a}$$

$$x = x(q,w) \tag{12b}$$

$$L = L(q,w) \tag{12c}$$

By substituting equations (12a), (12b) and (12c) into equation (2), and then taking expectations, we obtain the expected indirect utility function

$$E[V] = \Omega(q,w) + \varphi(g) + E[v(b\,x(q,w))] \tag{13}$$

The government chooses t, t and g such as to maximize the expected indirect utility function subject to the budget constraint. By using the short notation

$$R(t,t) = tx(q,w) + tw[H - L(q,w)],$$

the public sector budget constraint can be written as

$$R(t,t) = g \tag{14}$$

By substituting the budget constraint into the expected indirect utility function, we can write the optimization problem in a way similar to that of the previous section

$$MaxE[V] = \underset{t,t}{Max}\,\Omega(q,w) + j\,(R(t,t)) + E[v(b\,x(q,w))]$$

and the first order conditions can be written as

$$E[V_t] = \Omega_q + j_g R_t + x_q E[b\,v_z(bx)] = 0 \tag{15a}$$

$$E[V_t] = -\Omega_w w + j_g R_t - x_w wE[b\,v_z(bx)] = 0 \tag{15b}$$

We assume that the second order sufficient conditions for maximization are fulfilled. As in the previous section, the main concern is to analyze how a mean preserving increase in the spread of environmental damage affects the optimal environmental tax. Differentiating equations (15a) and (15b) with respect to t, t and q, we obtain

$$\begin{bmatrix} E[V_{tt}] & E[V_{tt}] \\ E[V_{tt}] & E[V_{tt}] \end{bmatrix} \begin{bmatrix} \partial t / \partial q \\ \partial t / \partial q \end{bmatrix} = \begin{bmatrix} -x_q \{ \partial E[b v_z (bx)] / \partial q \} \\ x_w w \{ \partial E[b v_z (bx)] / \partial q \} \end{bmatrix} \tag{16}$$

We can then solve for the two partial derivatives as follows;

$$\frac{\partial t}{\partial q} = \frac{-[x_w w E[V_{tt}] + x_q E[V_{tt}]]\{\partial E[b v_z (bx)] / \partial q\}}{B} \tag{17a}$$

$$\frac{\partial t}{\partial q} = \frac{[x_q E[V_{tt}] + x_w w E[V_{tt}]]\{\partial E[b v_z (bx)] / \partial q\}}{B} \tag{17b}$$

where $E[V_{tt}] < 0$, $E[V_{tt}] < 0$ and $B = E[V_{tt}]E[V_{tt}] - \{E[V_{tt}]\}^2 > 0$ according to the second order sufficient conditions for maximization. We have derived the following result with respect to the environmental tax;

Proposition 3: *With nondecreasing absolute risk aversion, an increase in q will increase (decrease) the optimal environmental tax if, and only if, $x_q E[V_{tt}] + x_w w E[V_{tt}]$ is positive (negative).*

The term $x_q E[V_{tt}]$ is unambiguously positive and contributes to increase the optimal environmental tax, as long as the assumption about nondecreasing absolute risk aversion applies. By analogy to the previous section, this effect can be thought of as reflecting the precautionary motive for environmental taxation. At the same time, an increase in q will also affect the optimal labor income tax which, in turn, influences the consumption of dirty goods. The sign of $x_w w E[V_{tt}]$ is ambiguous, since both x_w and $E[V_{tt}]$ can go in either direction. This makes the impact of q on t ambiguous as well. Therefore, even if nondecreasing absolute risk aversion provides an incentive to increase taxes on goods whose consequences for the environment are uncertain, it is not necessarily optimal to do so.

4. Discussion

This paper analyzes optimal tax policy in a model, where the relationship between consumption and environmental damage is uncertain. The main purpose is to examine how additional uncertainty about this relationship affects the optimal environmental tax, where the latter is defined as a unit tax on the consumption good that is causing damage to the environment. As it turns out, the results depend primarily on (i) the attitude towards risk and (ii) how the other tax instruments affect the consumption good that is causing environmental damage. If the private agents do not become less risk averse when the environmental damage increases - meaning that the preferences are characterized by nondecreasing absolute risk aversion - there is a precautionary motive for environmental taxation. This effect will work to increase the optimal environmental tax as a response to additional uncertainty. However, the influence of other tax instruments may offset the precautionary motive to increase the environmental tax. Therefore, even if nondecreasing absolute risk aversion provides an incentive to increase taxes on such goods, whose consequences to the environment are uncertain, it may not be optimal to do so in an economy with several tax instruments and other policy objectives than internalizing externalities.

Appendix

Proof of Lemma 1:

Since $b = \bar{b} + qe$, we can define

$$\frac{\partial E[bv_z(bx)]}{\partial q} = E[ev_z(bx)] + \bar{b}xE[ev_{zz}(bx)] + qxE[e^2 v_{zz}(bx)] \quad \text{(A1)}$$

The first term on the right hand side of equation (A1), $E[ev_z(bx)] = \text{cov}(e, v_z)$, is negative, because $dv_z(\cdot)/de = q\, x v_{zz}(\cdot) < 0$. Similarly, the third term on the right hand side is nonpositive, since $q > 0$, $x > 0$, $e^2 \geq 0$ and $v_{zz}(\cdot) < 0$.

To sign the second term on the right hand side of equation (A1), let us define $z^0 = \bar{b}x$. Consider first the case when $z > z^0$, so $e > 0$. Nondecreasing absolute risk aversion then implies

$$\frac{v_{zz}(z)}{v_z(z)} \geq \frac{v_{zz}(z^0)}{v_z(z^0)} \tag{A2}$$

or, if we multiply both sides by $v_z(z) < 0$,

$$v_{zz}(z) \leq \frac{v_{zz}(z^0)}{v_z(z^0)} v_z(z) \tag{A3}$$

Multiplying equation (A3) by $e > 0$ and then taking expectations on both sides, we find

$$E[ev_{zz}(z)] \leq \frac{v_{zz}(z^0)}{v_z(z^0)} E[ev_z(z)] < 0 \tag{A4}$$

If, on the other hand, $z < z^0$ and $e < 0$, the equivalent of equation (A3) takes the form

$$v_{zz}(z) \geq \frac{v_{zz}(z^0)}{v_z(z^0)} v_z(z) \tag{A5}$$

Finally, by multiplying equation (A5) by $e < 0$ and then taking expectations, we will again obtain equation (A4).

Therefore, $E[ev_{zz}(bx)] \leq 0$ and the second term on the right hand side of equation (A1) is nonpositive.■

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