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Family Job Search, Wage Bargaining, and Optimal Unemployment Insurance

Susanne Ek* Bertil Holmlund†

*Uppsala, susanne.ek@nek.uu.se

†Uppsala, bertil.holmlund@nek.uu.se

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Family Job Search, Wage Bargaining, and Optimal Unemployment Insurance*

Susanne Ek and Bertil Holmlund

Abstract

In this paper we develop an equilibrium search and matching model where two-person families as well as singles participate in the labor market. We show that equilibrium entails wage dispersion among equally productive risk-averse workers. Marital status as well as spousal labor market status matters for wage outcomes. In general, employed members of two-person families receive higher wages than employed singles. The model is applied to a welfare analysis of alternative unemployment insurance systems, recognizing the role of spousal employment as a partial substitute for public insurance. The optimal system involves benefit differentiation based on marital status as well as spousal labor market status.

KEYWORDS: wage bargaining, wage differentials, unemployment, unemployment insurance

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1 Introduction

The literature on job search has largely ignored the fact that around every second labor force participant is a member of a multiple-person household. This stands in stark contrast to the literatures on consumption, labor supply and income distributions, where the family perspective is paramount. In this paper we develop an equilibrium search and matching model of the labor market where the family perspective stands in focus. Marital status and the possibility of spousal income sharing generally matter for wage outcomes when individuals are risk averse and wages are determined through bargaining between workers and firms. In fact, our model implies wage differentials among equally productive workers.

"Large" multiple-person families do feature in some realms of equilibrium search and matching theory. A seminal contribution in this genre was offered by Merz (1995) who studied an economy where each household was described as "a very large extended family", whose members could perfectly insure each other against fluctuations in labor income associated with transitions between employment and unemployment. This approach has been adopted by others, including Hall and Milgrom (2008) in a recent paper. The "large family" approach has its virtues, but realism is not one of them. Modern industrialized economies are largely based on husband-wife families with at most two adult workers. Transfers across generations may occur so as to achieve some income smoothing, but complete smoothing is clearly unrealistic. Empirical work has documented that consumption among U.S. workers falls when unemployment strikes and that the presence of unemployment insurance markedly reduces the drop in consumption (Gruber, 1997).[1] Browning and Crossley (2001, 2009) report qualitatively similar results in studies of Canadian workers, especially for those with no liquid assets when entering into unemployment.

As far as we can tell, the model we propose is new in the literature. The economy we study is populated by two types of households, singles and couples. All household members participate in the labor force and are either employed or unemployed. Wages are set in a decentralized fashion through worker-firm bargaining. An unemployed worker in a two-person family can benefit from some consumption insurance through a working spouse, an option that is not available for singles. This will generally lead to different bargaining outcomes for singles and couples since the outside options differ. Wages will also differ between *ex ante* identical members of different two-person families depending on whether the family has one or two employed members.

[1] Gruber (1997) argues that the empirical results "decisively reject the notion that there are complete private consumption markets for unemployment spells..."

Our model naturally lends itself to an analysis of optimal unemployment insurance (UI). The availability of some private income smoothing should arguably be recognized when designing the optimal UI system. One issue is whether benefits should be based on individual or family income. When wages differ across workers, the question of optimal replacement rates also becomes relevant. For example, does the optimal system involve flat rate or earnings-related benefits? This issue is related to the debate on "Bismarckian" versus "Beveridgean" social insurance schemes; see for example Casamatta et al. (2000) and Goerke (2000).

The paper proceeds with a brief discussion of the related literature. The model is introduced in Section 3. We show that marital status as well as spousal labor market status matter for wage outcomes. Section 4 provides a welfare analysis of alternative unemployment insurance systems, while recognizing the role of spousal employment as a partial substitute for public insurance. Section 5 concludes the paper.

2 Related Literature

The paper relates to the literature on wage dispersion in frictional labor markets. The empirical literature has documented that wages vary among workers with observationally similar characteristics, a fact that has inspired modeling of frictional wage dispersion. This literature, mainly in the search and matching tradition, has derived conditions under which wage differentials can arise even for workers who are *ex ante* identical, i.e., identical before labor force status is determined. The wage-posting model of Burdett and Mortensen (1998) is a leading example.[2] In their model, job seekers are *ex ante* identical but may end up with different reservation wages as they have the option of engaging in on-the-job-search and job-to-job mobility. This reservation wage heterogeneity creates a trade-off for firms: high-wage firms are able to attract and retain more workers than low-wage firms are, but the rent per worker that high-wage firms can extract is relatively low. Our paper shows that wage differentials can arise among workers with identical *ex ante* characteristics, such as between workers in two different two-person families where a partner is unemployed in one case and employed in the other. Spousal employment outcomes matter for bargained wages when risk-averse spouses practice income sharing.

Another related literature is concerned with the "added worker effect" and spousal labor supply as insurance. Burdett and Mortensen (1978) study labor supply under uncertainty and consider job search by couples using a standard partial

[2] See Rogerson et al. (2005) for a survey of search models of the labor market.

equilibrium search framework. When one family member becomes unemployed, part of the income loss can be offset by increased spousal labor supply.[3] As shown by Cullen and Gruber (2000), this supply response may be substantially weakened by unemployment insurance. Our paper assumes exogenous search intensity and spousal job loss leads to wage adjustment but no change in search effort. However, an extension of the basic model to incorporate endogenous search effort would include mechanisms akin to the added worker effect. An unemployed family member's search effort would respond to labor market outcomes of the spouse since such outcomes influence overall family income.

A recent related paper is Guler et al. (2010) which extends the canonical partial equilibrium single-agent search model by studying joint search of couples who practice income pooling. In particular, the family's reservation wage policies are characterized. The reservation wage of an unemployed spouse depends on the wage of the working spouse and acceptance of a job offer may trigger a quit decision by the working partner. The wage offer distribution is taken as exogenous but accepted wages are determined by the couple's joint search. Our paper ignores wage search but features endogenous wage determination through worker-firm bargaining. As in Guler et al. (2010), the labor market fortunes of one spouse will generally affect wage outcomes for the other spouse, but the wage adjustment is driven by bargaining incentives rather than by search incentives. Empirical research on how spousal employment and income changes affect wage outcomes is meager.

Our paper also relates to the literature on optimal unemployment insurance design. This literature has focused on issues such as the case for benefit variation over the spell of unemployment and the interaction between UI and active labor market policy.[4] Most papers have considered economies without wage dispersion. However, when wages differ across workers, a new issue arises concerning the optimal differentiation of benefits across workers with different past or prospective wages. One policy, in the spirit of Beveridge, involves flat rate benefits. A "Bismarckian" alternative instead involves earnings-related benefits, thus implying higher benefit levels for workers with high past or potential wages. We also consider benefit differentiation based on marital status and spousal labor market status. The model is used to provide welfare assessments of alternative UI systems.

Finally, we note that numerous empirical studies have documented the existence of a male marriage wage premium: married males earn substantially higher wages than unmarried men (see e.g. Korenman and Neumark, 1991). Our paper suggests that a marriage premium can arise as a result of wage bargaining when

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[3] Garcia-Perez and Rendon (2004) present an empirical model of family job search and consumption where spousal interactions are modeled in detail, partly along the lines of Burdett and Mortensen (1978).

[4] See Fredriksson and Holmlund (2006) for a survey.

married persons can effectively wield stronger bargaining power as a result of intra-family income pooling. However, our model does not explain why marriage appears to yield a wage premium for men but not for women.

3 The Model

3.1 The Labor Market

The economy is populated by households that are either singles or couples. In the following, we sometimes refer to members of two-person families as husbands and wives. The individuals have identical preferences and are equally productive in all firms. All individuals are labor force participants. The total labor force is fixed and normalized to unity. There are no transitions between marital states so the fraction of singles in the labor force is constant.[5] Workers are either employed or unemployed and have infinite time horizons. Time is continuous and an employed worker is separated from her job at an exogenous Poisson rate ϕ. Upon entering unemployment, the worker is immediately eligible for (time-invariant) UI benefits.

There are three relevant labor market states for a two-person family: (i) both spouses employed; (ii) both spouses unemployed; and (iii) one spouse employed and one spouse unemployed. A family where both spouses are (un)employed is referred to as fully (un)employed; a family with mixed employment status is referred to as partially (un)employed.

All unemployed workers are engaged in job search with an exogenous intensity. There is no on-the-job search. The matching function that relates the aggregate flow of hires to the number of vacancies (v) and the number of unemployed (u) exhibits constant returns to scale: $M = m(v,u)$. Let $\theta \equiv v/u$ denote labor market tightness. The probability per unit time that an individual finds a job is $\alpha = m(v,u)/u = \alpha(\theta)$. Moreover, $\alpha(\theta) = m(v,u)/u = m(\theta,1)$ and hence $\alpha'(\theta) > 0$; the tighter the labor market, the easier it is to find a job. Firms fill vacancies at the rate $q(\theta) = m(v,u)/v = m(1,1/\theta)$, and thus $q'(\theta) < 0$; the tighter the labor market, the more difficult it is to fill a vacancy. By constant returns to scale, $\alpha(\theta) = \theta q(\theta)$ holds.

The steady state flow equilibrium relationship for this economy can be summarized by an unemployment relationship of the form

$$u = \frac{\phi}{\phi + \alpha(\theta)} \tag{1}$$

[5] We treat marital status as exogenous for tractability reasons and in order to focus on incentives in wage bargaining.

This is the aggregate unemployment rate in the economy as well as the unemployment rate pertaining to singles and couples, respectively. Absent differences in search efforts or separation rates across groups, there will be no group differences in unemployment rates (or, equivalently, the fraction of time spent as unemployed). The probability that any given individual is unemployed is thus given by u and the employment probability is $1-u$. The spouses' probabilities of being (un)employed are independent of each other. The probability that a couple is fully employed is thus $(1-u)^2$, the probability of a mixed employment status is $2u(1-u)$, and the probability that both spouses are unemployed is u^2.

3.2 Households

The individual's instantaneous utility function is increasing in consumption. Individuals do not have access to a capital market so consumption equals income at each instant. Couples practice income sharing at a 50/50 rate, whereby each spouse receives half of the total family income. All incomes, including benefits, are taxed at the rate t. The level of consumption varies across individuals in two dimensions, labor market status (employed vs. unemployed) and marital status (single vs. couple). For employed singles, average household after-tax income is simply the net wage, i.e., $\tilde{w}_0 \equiv w_0(1-t)$. For unemployed singles, net income is given by after-tax unemployment benefits, $\tilde{b} \equiv b(1-t)$. For couples, there are three possibilities depending on labor market status. If both spouses are unemployed, average net income is $(\tilde{b}+\tilde{b})/2 = \tilde{b}$; if one spouse is employed and the other is unemployed, average net income is $(\tilde{w}_1+\tilde{b})/2$, where $\tilde{w}_1 \equiv w_1(1-t)$ is the net wage received by the working spouse; if both spouses are employed, average net income per member is given by $(\tilde{w}_2+\tilde{w}_2)/2 = \tilde{w}_2$, where $\tilde{w}_2 \equiv w_2(1-t)$ is the net wage received by members of a fully employed family. The logic of the wage notation for couples is that subscript 1 is used if one person is employed and subscript 2 if two persons are employed.

Wages may differ with respect to marital status and may also differ depending on whether workers belong to fully or partially employed families. As will be shown, such wage differentials may arise under Nash bargaining over wages. For now we proceed under the assumption that benefits are of the flat rate variety: all unemployed individuals thus receive the same benefit level, b, when unemployed. This may not be an optimal UI system, an issue to which we will return.

Utility functions are taken to be isoelastic of the form

$$v(c) = \frac{c^{1-\rho}-1}{1-\rho}$$

where c denotes consumption (income) and ρ is the degree of relative risk aversion, $\rho \geq 0$. Linear utility obtains when $\rho = 0$ and logarithmic utility when $\rho \to 1$. The notation for the instantaneous utilities in the various states are as follows: unemployed singles as well as unemployed individuals in wholly unemployed families, $\upsilon(\tilde{b})$; employed singles, $\upsilon(\tilde{w}_0)$; members of partially employed families, $\upsilon(\tilde{w}_1;\tilde{b})$ or simply $\upsilon(\tilde{w}_1)$ (recall that average net income in this state is $(1-t)(w_1+b)/2$); and members of wholly employed families, $\upsilon(\tilde{w}_2)$.

Consider the intertemporal objective functions for singles (superscript s) and couples (superscript c). Let U^s denote the expected discounted present value of utility for a single unemployed worker and let N^s denote the corresponding value if the person is employed. The value functions can be written as

$$rU^s = \upsilon(\tilde{b}) + \alpha(N^s - U^s) \tag{2}$$

$$rN^s = \upsilon(\tilde{w}_0) + \phi(U^s - N^s) \tag{3}$$

where $\alpha = \alpha(\theta)$ and r is the subjective rate of time preference. These two equations imply a present value differential between employment and unemployment of the form

$$N^s - U^s = \frac{\upsilon(\tilde{w}_0) - \upsilon(\tilde{b})}{\alpha + \phi} \tag{4}$$

when evaluated at $r = 0$.

For individuals living in two-person families, the value functions are slightly non-standard since income sharing implies that the spousal valuations are interdependent: labor market events directly affecting the husband affect the wife's consumption, and vice versa. Let U^c denote the expected discounted present value of utility for each family member if both spouses are unemployed, E^c the value associated with mixed employment status, and N^c the corresponding value if both spouses are employed. There are three relevant value functions for a member of a two-person household:

$$rU^c = \upsilon(\tilde{b}) + 2\alpha(E^c - U^c) \tag{5}$$

$$rE^c = \upsilon(\tilde{w}_1;\tilde{b}) + \alpha(N^c - E^c) + \phi(U^c - E^c) \tag{6}$$

$$rN^c = \upsilon(\tilde{w}_2) + 2\phi(E^c - N^c) \tag{7}$$

Consider the case where two family members are unemployed. The husband (as well as the wife) receives $\tilde{b}$ as instantaneous unemployment compensation. He finds a job at the rate α, thereby entering partial employment and its associated present value E^c. The capital gain from such a transition is $E^c - U^c$. His wife also finds a job at the rate α and her transition takes her, as well as her husband, to partial employment with present value E^c. The probability that both spouses simultaneously receive job offers is negligible in a short time interval. Consider next the case with mixed employment status (partial unemployment). An unemployed husband finds a job at the rate α, a transition that is associated with present value N^c. His employed wife runs the risk ϕ of losing her job, thereby moving the family into full unemployment with present value U^c. Finally, the fully employed household includes spouses who both earn $\tilde{w}_2$. The husband as well as the wife runs the risk ϕ of being laid off, thus entering partial unemployment. The probability that both spouses will simultaneously be laid off is negligible in a short time interval.

Evaluated at $r = 0$, the present value differences can be written as:

$$E^c - U^c = \left(\frac{1}{\alpha+\phi}\right)\frac{(\alpha+2\phi)\left[v(\tilde{w}_1)-v(\tilde{b})\right]+\alpha\left[v(\tilde{w}_2)-v(\tilde{w}_1)\right]}{2(\alpha+\phi)} \quad (8)$$

$$N^c - E^c = \left(\frac{1}{\alpha+\phi}\right)\frac{(2\alpha+\phi)\left[v(\tilde{w}_2)-v(\tilde{w}_1)\right]+\phi\left[v(\tilde{w}_1)-v(\tilde{b})\right]}{2(\alpha+\phi)} \quad (9)$$

Consider eq. (8). The present value difference between partial and full unemployment is the discounted value of a weighted average of utility differences between partial and full unemployment, $v(\tilde{w}_1) - v(\tilde{b})$, and between full and partial employment, $v(\tilde{w}_2) - v(\tilde{w}_1)$. The weights depend on the job finding rate, α, and the job destruction rate, ϕ. Note that the value difference $E^c - U^c$ is more heavily affected by the immediate income difference between partial and full unemployment, $v(\tilde{w}_1) - v(\tilde{b})$, than by the prospective future income difference between full and partial employment, $v(\tilde{w}_2) - v(\tilde{w}_1)$. Analogous interpretations hold for eq. (9).

With log utility, it is obvious that the utility differences in (4), (8) and (9) are independent of the tax rate. That is, $v(\tilde{w}_0) - v(\tilde{b}) = v(w_0) - v(b)$, $v(\tilde{w}_1) - v(\tilde{b}) = v(w_1) - v(b)$ and $v(\tilde{w}_2) - v(\tilde{w}_1) = v(w_2) - v(w_1)$. It follows that the present value differences given by (4), (8) and (9) are also invariant to changes in the tax rate. In the general case we have:

$$v(\tilde{w}_0) - v(\tilde{b}) = \frac{1}{1-\rho}\left[w_0^{1-\rho} - b^{1-\rho}\right](1-t)^{1-\rho} \quad (10)$$

$$v(\tilde{w}_1) - v(\tilde{b}) = \frac{1}{1-\rho}\left[\left(\frac{w_1+b}{2}\right)^{1-\rho} - b^{1-\rho}\right](1-t)^{1-\rho} \quad (11)$$

$$v(\tilde{w}_2) - v(\tilde{w}_1) = \frac{1}{1-\rho}\left[w_2^{1-\rho} - \left(\frac{w_1+b}{2}\right)^{1-\rho}\right](1-t)^{1-\rho} \quad (12)$$

Multiple person households can benefit from economies of scale in consumption, a possibility that is recognized in studies of income distribution among households of different sizes. The literature has suggested several alternative "equivalence scales" so as to allow welfare comparisons across different family sizes (see Atkinson et al., 1995). The square root scale is one example. This scale divides household income by the square root of household size. To get individual size-adjusted income for a two-person family we would thus divide family income by $\sqrt{2} \approx 1.4$ rather than by 2. Such an adjustment does not affect equilibrium outcomes or optimal policies in our model. The reason is that the square root adjustment applies to all incomes, benefits as well as wages. The adjustment is in fact analogous to a subsidy to couples that is proportional to income. With log utility, the instantaneous utility differences in (8) and (9) will be independent of the square root factor in the same way as they are independent of the tax rate. It follows that the present value differences given by (8) and (9) are also unaffected by this factor. In the general isoelastic case, we obtain expressions analogous to (11) and (12).

3.3 Firms

Firms operate under constant returns to labor, an assumption that allow us to treat a job as a stand in for the firm (Pissarides, 2000). Workers and jobs are randomly matched, implying that the firm with some probability will encounter a worker from a single-person household, a worker from a wholly unemployed family, or a worker from a partially employed family. These three categories of workers may earn different wages. Let y denote the constant level of labor productivity, uniform across firms and workers, J_0 the present discounted value of a job occupied by a single-household worker, J_1 the value of a job occupied by a worker from a partially employed family, and J_2 the value of a job matched to a member of a wholly employed family. The value of opening a vacancy is denoted V. The value functions pertaining to occupied jobs are written as:

$$rJ_0 = y - w_0 + \phi(V - J_0) \tag{13}$$

$$rJ_1 = y - w_1 + \phi(V - J_1) + \alpha(J_2 - J_1) \tag{14}$$

$$rJ_2 = y - w_2 + \phi(V - J_2) + \phi(J_1 - J_2) \tag{15}$$

where r here stands for the rate of interest, by assumption equal to the individual's subjective rate of time preference.

Eq. (13) is the standard job valuation function with one type of worker and no wage differences. The firm's instantaneous surplus is given by $y - w_0$ and the job is destroyed at the exogenous rate ϕ. Eqs. (14) and (15) are non-standard and capture worker interdependencies in two-person families. Consider eq. (14). A job occupied by a worker from a partially employed family is destroyed at the rate ϕ, as is a job occupied by a worker from a single-person household. However, there is also a possibility that the worker's unemployed spouse will find a job, an event that triggers a wage renegotiation. The spouse encounters and accepts job offers at the rate α, causing a change in the present value of the job equal to $J_2 - J_1$. The third value function, eq. (15), states that the value of a job occupied by a worker from a wholly employed family runs two types of risks. There is a risk that the job itself is destroyed, an event that occurs at the rate ϕ. There is also a risk that job desctruction occurs in the firm of the worker's spouse; this event also strikes at the rate ϕ. This spousal job loss leads to wage renegotiation and therefore a change in the value of the job.

The solutions of the value functions, evaluated at $r = 0$ and $V = 0$ (free entry) are obtained as

$$J_0 = \frac{1}{\phi}(y - w_0) \tag{16}$$

$$J_1 = \frac{1}{\phi}\left[y - (\lambda w_1 + (1-\lambda)w_2)\right]; \quad \lambda \in (0,1) \tag{17}$$

$$J_2 = \frac{1}{\phi}\left[y - (\delta w_1 + (1-\delta)w_2)\right]; \quad \delta \in (0,1) \tag{18}$$

where $\lambda \equiv 2\phi/(\alpha + 2\phi)$, $\delta \equiv \phi/(\alpha + 2\phi)$, and $J_2 - J_1 = (w_1 - w_2)/(\alpha + 2\phi)$. The value of an occupied job is given as the discounted present value of the surplus. Note that the average wage cost pertaining to employed couples is given as a weighted average of the wages for members of partially and wholly employed families, w_1 and w_2.

It remains to consider the value of opening a vacancy. The flow value of keeping a vacancy is denoted k and the firm meets unemployed job seekers at the rate $q(\theta)$. The probability that a job seeker is single is given by the fraction of singles in the population, γ; the probability of finding a married seeker is thus $1-\gamma$. Upon encountering a married job seeker, the probability that he or she belongs to a wholly unemployed family is u whereas the probability of coming from partial employment is $1-u$. The value function takes the form

$$rV = -k + q(\theta)\left[\gamma J_0 + (1-\gamma)(uJ_1 + (1-u)J_2) - V\right] \tag{19}$$

which can be rewritten as

$$y - \{\gamma w_0 + (1-\gamma)[uw_1 + (1-u)w_2]\} = \frac{k\phi}{q(\theta)} \tag{20}$$

where free entry, $V = 0$, is imposed along with eqs. (16), (17) and (18). Note also that $u = u(\theta)$ as given by (1). The left-hand side of (20) is the excess of the marginal product of labor over expected wage costs. In equilibrium, this surplus equals the expected capitalized value of the vacancy cost, i.e., $k\phi/q(\theta)$. Since $u'(\theta) < 0$ and $q'(\theta) < 0$, the job creation condition provides a relationship between tightness and each of the three wage rates. A sufficient (but not necessary) condition for $\partial\theta/\partial w_j < 0,\ j = 0, 1, 2,$ is $w_2 \geq w_1$.

3.4 Wage Bargaining

Wages are determined by decentralized worker-firm Nash bargaining. As usual in these models, the relevant threat point for the single worker is the value of unemployment, U^s. Let $\beta \in (0, 1)$ denote the worker's bargaining power. The relevant Nash product for singles is then

$$\Omega(w_0) \equiv (N^s - U^s)^\beta (J_0 - V)^{1-\beta}$$

and the first-order condition evaluated at $V = 0$ is

$$(1-\beta)(N^s - U^s) = \beta J_0 \frac{\partial v(\tilde{w}_0)}{\partial w_0} \tag{21}$$

We note that this condition is independent of the tax rate. For log utility, both the left-hand side and the right-hand side are independent of the tax rate. In the general isoelastic case, each side is multiplied by a tax factor, $(1-t)^{1-\rho}$, which drops out from (21).

For workers in two-person families, there are two cases to consider. A worker from a partially employed family has continued unemployment as the relevant threat point, i.e., U^c. The relevant Nash product is thus

$$\Omega(w_1) \equiv (E^c - U^c)^\beta (J_1 - V)^{1-\beta}$$

with the first-order condition at $V = 0$ given by

$$(1-\beta)(E^c - U^c) = \beta J_1 \frac{\partial v(\tilde{w}_1)}{\partial w_1} \tag{22}$$

Again we note that the first-order condition is independent of the tax rate. For log utility, both the left-hand side and the right-hand side are independent of the tax rate. In the general isoelastic case, both sides are multiplied by a tax factor which drops out from (22).

The threat point for a worker from a wholly employed family is different since income sharing cushions the income loss associated with failure to strike a bargain. We assume that each spouse acts on her own, taking the partner's wage, $\hat{w}_2$, as given. Instantaneous income associated with disagreement is given by $\left(\tilde{b}+\tilde{w}_1\right)/2$ rather than $\tilde{b}$ and the relevant threat point is thus given by E^c. The Nash product is then

$$\Omega(w_2) \equiv (N^c - E^c)^{\beta} (J_2 - V)^{1-\beta}$$

and the corresponding first-order condition at $V = 0$ is

$$(1-\beta)\left(N^c - E^c\right) = \beta J_2 \frac{\partial v(\tilde{w}_2;\hat{w}_2)}{\partial w_2} \tag{23}$$

where the condition is independent of the tax rate as in the previous first-order conditions.[6]

The magnitude of a worker's instantaneous marginal utility of a wage increase, $\partial v(\tilde{w}_j)/\partial w_j$, $j = 0,1,2$, plays a crucial role for wage outcomes. For a single worker, we have $\partial v(\tilde{w}_0)/\partial w_0 = 1-t$ for linear utility and $\partial v(\tilde{w}_0)/\partial w_0 = 1/w_0$ for log utility. For a worker in a partially employed family, we get $\partial v(\tilde{w}_1)/\partial w_1 = (1/2)(1-t)$ for linear and $\partial v(\tilde{w}_1)/\partial w_1 = 1/(b+w_1)$ for log utility. Finally, for workers in wholly employed families we have $\partial v(\tilde{w}_2)/\partial w_2 = (1/2)(1-t)$ for linear and $\partial v(\tilde{w}_2)/\partial w_2 = 1/2w_2$ for log utility when evaluated at a symmetric equilibrium with $w_2 = \hat{w}_2$. It is clear from the first-order conditions that an increase in the marginal utility of a wage hike is analogous to an increase in the worker's relative bargaining power, i.e., $\beta/(1-\beta)$.

3.5 Equilibrium

All the ingredients of the model are now in place. There are 11 endogenous variables: u, θ, $N^s - U^s$, $E^c - U^c$, $N^c - E^c$, J_0, J_1, J_2, w_0, w_1, w_2. The relevant equations are (1), (4), (8), (9), (16), (17), (18), (20) – (23). To solve the model it is useful to focus on the job creation condition along with the three wage bargaining equations. The latter three equations, stated in (21), (22) and (23) can, after relevant substitutions, be written as

[6] Note that accounting for economies of scale through a square root factor for two-person families does not affect bargained wages or any other endogenous variable. The reason is that the square root factor is analogous to a (negative) proportional income tax that applies to benefits as well as wages.

$$\frac{1}{\alpha+\phi}\left[v(\tilde{w}_0)-v(\tilde{b})\right] = \hat{\beta}\left(\frac{y-w_0}{\phi}\right)\frac{\partial v(\tilde{w}_0)}{\partial w_0} \tag{24}$$

$$\frac{\alpha\, v(\tilde{w}_2)+2\phi\, v(\tilde{w}_1)-(\alpha+2\phi)\, v(\tilde{b})}{2(\alpha+\phi)^2} = \frac{\hat{\beta}}{\phi}\left[y-\lambda w_1-(1-\lambda)w_2\right]\frac{\partial v(\tilde{w}_1)}{\partial w_1} \tag{25}$$

$$\frac{(\phi+2\alpha)\, v(\tilde{w}_2)-2\alpha\, v(\tilde{w}_1)-\phi\, v(\tilde{b})}{2(\alpha+\phi)^2} = \frac{\hat{\beta}}{\phi}\left[y-\delta w_1-(1-\delta)w_2\right]\frac{\partial v(\tilde{w}_2;\hat{w}_2)}{\partial w_2} \tag{26}$$

where $\hat{\beta}\equiv\beta/(1-\beta)$ measures the worker's relative bargaining power, $\lambda\equiv 2\phi/(\alpha+2\phi)$, $\delta\equiv\lambda/2$ and $\alpha=\alpha(\theta)$. Eqs. (24) – (26), and assuming a symmetric equilibrium with $w_2=\hat{w}_2$ along with the job creation condition (20), determine $\theta,\ w_0,\ \ w_1$ and w_2. Unemployment is obtained from (1) once tightness is determined. The numerical versions of the model that we have considered always deliver unique equilibria.[7]

As we have noted, the bargained (gross) wages are independent of the tax rate; the tax is thus completely borne by workers and the tax rate can be determined residually from the government's budget restriction. Government revenues are obtained from

$$\begin{aligned} T \equiv\ & t\,(1-u)\{\gamma\, w_0+(1-\gamma)\,[u w_1+(1-u)w_2]\} \\ & +t\{\gamma\, ub+(1-\gamma)\, u\,[(1-u)\, b+ub]\} \end{aligned}$$

and benefit expenditures are given by

$$B\equiv\gamma\, ub+(1-\gamma)u\,[(1-u)b+ub]$$

The tax rate is thus obtained via the budget restriction $T(t)=B$.

3.6 Wage Differentials

Consider wage outcomes for the three types of workers, viz. a worker from a single-person household, a worker from a partially employed family, and a worker from a wholly employed family. It is useful to begin with linear utility functions, in which case we obtain closed form solutions for the wage equations, i.e., bargained wages

[7] With isoelastic utility and risk aversion ($\rho>0$), $b>0$ is required.

as functions of (endogenous) tightness and the exogenous variables. The first-order conditions imply wage equations of the form

$$w_j = \frac{\beta\,[\alpha\,(\theta)+\phi]}{\phi+\alpha\,(\theta)\,\beta}y + \frac{(1-\beta)\,\phi}{\phi+\alpha\,(\theta)\,\beta}b \qquad j = 0, 1, 2 \tag{27}$$

Bargained wages are given as weighted averages of productivity and benefits. The weight on productivity is increasing in tightness, which implies that wages are increasing in tightness since $y > b$. It is immediately obvious that wages are independent of marital status and spousal labor market status. Hence:

Proposition 1 *Equilibrium in the family search model entails no wage dispersion among equally productive risk-neutral workers.*

To get some intuition for the role of risk aversion, consider a special case of our model where one spouse (the "breadwinner") is permanently employed and earns the wage ω, whereas the other spouse (the "secondary worker") moves back and forth between employment and unemployment. The breadwinner's wage effectively functions as a state-independent income subsidy to the secondary worker in the family. The instantaneous utility difference between employment and unemployment for this family would take the form

$$D \equiv \frac{(1-t)^{1-\rho}}{1-\rho}\left[(w+\omega)^{1-\rho} - (b+\omega)^{1-\rho}\right] \tag{28}$$

where $\partial D/\partial\omega \leq 0$ as $\rho \geq 0$ and $\omega > b$. With log utility we have $D = \ln(w+\omega) - \ln(b+\omega)$ and $\partial D/\partial\omega = (w+\omega)^{-1} - (b+\omega)^{-1} < 0$. A higher breadwinner wage reduces the utility difference between employment and unemployment when the utility function is strictly concave. This implies a decrease in the surplus from agreement and thus stronger incentives for a wage increase.

However, the impact of the subsidy on the negotiated wage depends also on the marginal utility of a wage hike, i.e., $\partial v(\tilde{w})/\partial w = (1-t)^{1-\rho}(w+\omega)^{-\rho}$. This marginal utility is decreasing in ω for $\rho > 0$. A higher breadwinner wage thus entails incentives for wage moderation via this mechanism; there is obviously zero impact if $\rho = 0$. With log utility, the wage moderation effect is dominated by the wage push effect, i.e., $\partial\tilde{w}/\partial\omega > 0$.[8] The bottom line is that state-independent

[8]To verify this claim, consider the first-order condition

$$\frac{1}{\alpha+\phi}\left[\ln(w+\omega) - \ln(b+\omega)\right] = \hat{\beta}\left(\frac{y-w}{\phi}\right)\frac{1}{w+\omega}$$

and differentiate with respect to w and ω, holding $\alpha(\theta)$ constant.

income supplements matter for wage outcomes when individuals are risk averse but they do not matter under risk neutrality.

Wage outcomes in our model are driven by the worker's surplus from agreement as well as by the marginal utility to the worker of a wage hike, $\partial v(\tilde{w}_j)/\partial w_j$, $j = 0,1,2$. Consider log utility and assume for a moment that wage equality prevails, i.e., $w_0 = w_1 = w_2 = w$. We would then have $\partial v(\tilde{w}_0)/\partial w_0 = 1/w$ for singles, $\partial v(\tilde{w}_1)/\partial w_1 = 1/(b+w)$ for partially employed families, and $\partial v(\tilde{w}_2)/\partial w_2 = 1/2w$ for wholly employed families. Hence the following inequalities would apply (for $b < w$): $\partial v(\tilde{w}_0)/\partial w_0 > \partial v(\tilde{w}_1)/\partial w_1 > \partial v(\tilde{w}_2)/\partial w_2$. The marginal utility of a wage hike would be highest for a worker from a single-person household. Moreover, the marginal utility of a wage hike would be higher for partially employed families than for wholly employed families. All else equal, these inequalities would suggest $w_0 > w_1 > w_2$.

All else is, of course, not equal. The worker's surplus from a wage agreement varies by marital status and spousal labor market status. Singles cannot benefit from partial income insurance via spousal transfers, which may increase the utility difference between employment and unemployment and thereby reduce wage pressure. It appears difficult to give precise general characterizations of wage differentials, but it turns out that overall wage equality is incompatible with equilibrium:

Proposition 2 *Equilibrium in the family search model entails wage dispersion among equally productive risk-averse workers.*

Proof. Assume $w_0 = w_1 = w_2 = w$ and check whether an equilibrium exists under these assumptions. Use (24) and (25) to substitute out $y - w$ and define $R \equiv b/w$ as the replacement rate. The resulting expression boils down to

$$2[\ln(1+R) - \ln 2] = \ln R$$

which has no meaningful solution since $LHS > RHS$ for $R \in (0,1)$. We have $LHS \in (-2\ln 2, 0)$ and $RHS \in (-\infty, 0)$ for $R \in (0,1)$. ■

To examine spousal wage differences we proceed analogously. Assume $w_1 = w_2 = w$ and use (25) and (26) to obtain

$$2\left(\frac{u}{1-u}\right) = \frac{\ln R}{\ln(1+R) - \ln 2} - \frac{4}{1+R} \tag{29}$$

where we have invoked $\phi/\alpha(\theta) = u/(1-u)$. Wage equality between spouses is a theoretical possibility, but requires a very low replacement rate. The right-hand side of (29) must be positive but a positive sign obtains only for $R < 0.137$. Consistency with realistic unemployment rates in a range between 5 and 10 percent requires even lower replacement rates; $R \approx 0.10$ is required for an unemployment rate of 10 percent. The replacement rate must thus be set unrealistically low in order to generate spousal wage equality as well as realistic values of unemployment.

We have used a numerical version of the model to examine wage and employment outcomes when individuals are risk averse; see Appendix 1. The model is calibrated to data for the United States. The baseline calibration involves a log utility function and flat rate benefits are fixed at 30 percent of productivity, $b/y = 0.3$; this implies a replacement rate (relative to wages) slightly above 30 percent. Output per worker is normalized to unity, $y = 1$. The baseline features an unemployment rate of 6 percent which is the U.S. average over the past four decades.

Table 1 shows how wages and unemployment vary by relative risk aversion. Wages are ranked as $w_1 > w_2 > w_0$. Thus employed family members receive higher wages than employed singles, while workers in partially employed families earn more than workers in wholly employed families. Recall also that $w_1 > w_2$ would be the likely outcome if the marginal utility of a wage hike is higher for a partially employed family than for a wholly employed family. The magnitudes of the wage differentials increase as risk aversion increases. With relatively high risk aversion ($\rho = 2$), workers in partially employed two-person families earn 4 percent higher wages than working singles. Two-person families can provide partial income insurance to their members, a fact that contributes to higher wage pressure by reducing the utility surplus of employment relative to unemployment.

Table 1. The impact of risk aversion.

	$\rho = 0$	$\rho = 1/2$	$\rho = 1$	$\rho = 2$
w_0	0.983	0.979	0.973	0.957
w_1	0.983	0.993	0.998	0.998
w_2	0.983	0.981	0.980	0.977
$\ln(w_1/w_0)$	0	0.014	0.025	0.042
$\ln(w_2/w_0)$	0	0.002	0.007	0.021
$\ln(w_2/w_1)$	0	−0.012	−0.019	−0.021
u	0.066	0.063	0.060	0.053

A noteworthy feature of our model is that the shares of singles and couples in the population matter for unemployment and wage differentials. Singles have no access to spousal income insurance, thereby suggesting that unemployment would fall if the share of singles (γ) increases. Indeed, this is what is confirmed by the numerical analysis. An increase in γ from 0.3 to 0.7 leads to a decline in unemployment from 6.1 to 5.9 percent.

3.7 The Impact of Benefits

To understand how benefits affect wage outcomes, consider the three bargaining equations as given by (24), (25) and (26). As usual and as is clear from (24), a uniform rise in benefits reduces the single worker's surplus from agreement and this tends to lead to a wage increase. From (26) it follows that the same mechanism operates for workers in wholly employed families. The incentives are less clear, however, for workers in partially employed families. Examine eq. (25) and note that a benefit increase has several effects. First, there is the conventional effect operating via $\upsilon(\tilde{b})$ which reduces the worker's surplus from agreement and thus triggers an increase in wage pressure. Second, there are two effects operating through $\upsilon(\tilde{w}_1;\tilde{b})$: (i) a rise in b is akin to an in-work subsidy which increases average income and thus $\upsilon(\tilde{w}_1;\tilde{b})$ for the partially employed family, an effect which tends to offset part of the conventional impact; (ii) a rise in b also reduces the marginal utility of a wage increase through $\partial\upsilon(\tilde{w}_1;\tilde{b})/\partial w_1$, a fact that encourages wage moderation. All in all, there is no reason to expect that uniform benefit increases should have uniform wage effects. It is conceivable that benefit increases may encourage wage moderation among workers in partially employed families. This conjecture is confirmed by the numerical exercises reported in Table 2: w_1 decreases when b increases from 0.4 to 0.5 whereas w_0 and w_2 increase.

Table 2. The impact of uniform benefit changes.

	$b/y = 0.3$	$b/y = 0.4$	$b/y = 0.5$
w_0	0.973	0.978	0.982
w_1	0.998	0.999	0.998
w_2	0.980	0.982	0.985
$\ln(w_1/w_0)$	0.025	0.021	0.016
$\ln(w_2/w_0)$	0.007	0.004	0.002
$\ln(w_2/w_1)$	−0.019	−0.017	−0.014
u	0.060	0.064	0.068

For reasons discussed above, the impact of *selective* benefit changes is likely to be different from the results shown in Table 2. Let b_1 denote the benefit level for an unemployed individual in partially unemployed families (one unemployed person) and b_2 the benefit level for fully unemployed families (two unemployed persons). The value functions are slightly modified when benefits are differentiated; see Appendix 2. Table 3 shows some numerical comparative statics on selective benefit changes based on our calibrated model. When b_1 is varied, benefits for singles and fully unemployed families are fixed at the baseline value, $b_0 = b_2 = 0.3$. When b_2 is varied, the other benefit levels are analogously fixed. It is clear that an increase in b_1 leads to strong wage moderation among workers in partially employed families, reflecting the fact that b_1 is analogous to an in-work subsidy.[9]

Table 3. The impact of selective benefit changes.

	$b_1/y = 0.3$	$b_1/y = 0.5$	$b_2/y = 0.3$	$b_2/y = 0.5$
w_0	0.973	0.973	0.973	0.973
w_1	0.998	0.915	0.998	1.073
w_2	0.980	0.988	0.980	0.978
$\ln(w_1/w_0)$	0.025	−0.061	0.025	0.098
$\ln(w_2/w_0)$	0.007	0.015	0.007	0.005
$\ln(w_2/w_1)$	−0.019	0.076	−0.019	−0.093
u	0.060	0.061	0.060	0.061

4 Optimal Unemployment Insurance

4.1 Alternative UI Schemes

We proceed by applying our model to an analysis of optimal unemployment insurance. When wages differ among workers, issues concerning the optimal structure of benefits become interesting. For example, should higher wages motivate higher unemployment benefits?

Unemployment insurance schemes differ markedly across countries. One alternative, often referred to as Beveridgean, involves flat rate benefits, i.e., identical benefit levels for all unemployed individuals. Another system, known as Bismarckian, entails earnings-related benefits and thus higher benefit levels for individuals with higher pre-unemployment earnings. Existing schemes typically differ from the polar types in various ways. One prevalent scheme has borrowed features from both Beveridge and Bismarck and involves a fixed replacement rate up to an earnings threshold and a constant benefit level for earnings above this threshold. UI schemes also differ with regard to the treatment of family income and individual income as the basis for benefit levels.

Our model can be used to examine the welfare aspects of some of these issues. So far, we have mainly focused on flat rate benefits. Here, however, we also consider alternatives where benefit levels differ depending on previous earnings. One alternative is the Bismarckian scheme, i.e., with constant replacement rates. Another alternative involves optimal differentiation of benefit levels. Benefit differentiation may be based on marital status as well as spousal labor market status. An

[9] We note that $y = 1 < w_1$ holds in one case. However, $J_1 > 0$ still holds and what matters for the value of an occupied job is the average wage cost relative to productivity as given by eq. (14).

analysis of optimal UI design also requires explicit treatment of the taxes required to finance the benefits.

Notations and definitions are as follows. Flat rate benefits are denoted b (as before). The benefit level for singles who are unemployed is denoted by b_0, the benefit level for unemployed individuals in partially (un)employed families by b_1, and the benefit level for individuals in fully unemployed families by b_2. The Bismarckian scheme is then defined as

$$R = \frac{b_0}{w_0} = \frac{b_1}{w_2} = \frac{b_2}{w_1} \tag{30}$$

where R is the common replacement rate. The logic of our definition for couples is as follows. The immediate unemployment risk facing an individual in a fully employed family with wage w_2 is partial unemployment associated with benefit level b_1; the fraction of replaced income is thus b_1/w_2. The immediate unemployment risk facing an employed individual in a partially employed family with wage w_1 is the risk of entering the state of being fully unemployed and then receive b_2; the replacement rate is b_2/w_1.

The social welfare function is taken to be utilitarian. To simplify the analysis and allow comparisons of steady states without considering adjustment paths, we let the discount rate approach zero and obtain the objective function as a weighted average of individual per-period expected utilities. With log utility, social welfare takes the form

$$\begin{aligned}\Lambda &= \ln(1-t) + \gamma\,[u \ln b_0 + (1-u)\ln w_0] + \\ &\quad (1-\gamma)\left[(1-u)^2 \ln w_2 + u^2 \ln b_2\right] + \\ &\quad (1-\gamma)\left[2u(1-u)\ln\left(\frac{w_1+b_1}{2}\right)\right]\end{aligned}$$

where we have used the fact that spouses' probabilities of being (un)employed are independent of each other. The probability that any given individual is unemployed is u and the employment probability is $1-u$. The probability that a couple is fully employed is $(1-u)^2$; the probability that both spouses are unemployed is u^2; and the probability of a mixed employment status is $2u(1-u)$. The tax rate is determined from the government's budget restriction.

We compare five policies. The baseline case is $b/y = 0.3$. The second case involves optimal choice of flat rate benefits. The third case entails optimal choice of three benefit levels ($b_0,\ b_1,\ b_2$), subject to a replacement rate restriction of the form given by (30). In the fourth case, benefits are differentiated based on marital status, but without recognizing spousal labor market status. Singles thus receive b_0 whereas unemployed couples receive $b_1 = b_2 = b^c$. The fifth case involves optimal choice of three benefit levels ($b_0,\ b_1,\ b_2$), thus recognizing marital status as well as spousal labor market status, but without imposing a replacement rate restriction.

4.2 Numerical Results

The results are displayed in Table 4. The welfare effect of a specific UI regime is measured relative to the flat rate benchmark. It is expressed as the equivalent of a consumption tax that equalizes welfare across policy regimes. Let Λ^U represent welfare associated with the benchmark and Λ^A welfare associated with an alternative policy. The measure of the welfare gain of policy A relative to policy U is given by the value of the tax rate τ that solves $\Lambda^A[(1-\tau)w;\cdot] = \Lambda^U$. With logarithmic utility functions, we have $\Delta\Lambda \equiv \Lambda^A - \Lambda^U = -\ln(1-\tau) \approx \tau$. We also show welfare gains separately for singles ($\Delta\Lambda^s$) and couples ($\Delta\Lambda^c$), where

$$\begin{aligned}\Lambda^s &= \ln(1-t) + u\ln b_0 + (1-u)\ln w_0 \\ \Lambda^c &= \ln(1-t) + (1-u)^2 \ln w_2 + u^2 \ln(b_2) + 2u(1-u)\ln\left(\frac{w_1 + b_1}{2}\right)\end{aligned}$$

and $\Delta\Lambda = (\Delta\Lambda^s + \Delta\Lambda^c)/2$.

The optimal flat benefit level (column 2) implies $b/y = 0.57$ and thus replacement rates close to 60 percent. The welfare gain relative to the baseline is relatively large, amounting to almost one percent of consumption. It is noteworthy that welfare gains for singles account for the overall welfare gain; in fact, couples experience a slight fall in welfare relative to the baseline with much less generous benefits whereas the welfare increase for singles amounts to almost two percent of consumption. Unemployment increases by approximately one percentage point and the tax rate increases by two percentage points.

A replacement rate restriction leads to a slight decrease in welfare compared to flat rate benefits (column 3). Benefit differentiation with respect to marital status (column 4) implies that singles should receive higher benefits than couples: the optimal system involves a replacement rate of 67 percent for singles and 43 percent for couples. Since singles do not have access to partial income insurance, it is to be expected that they should receive higher benefits than couples. Differentiation with respect to marital status also entails a substantial welfare gain for singles amounting to 3.6 percent of consumption relative to the baseline and 1.7 percent relative to the optimal flat rate policy. However, there is also a large welfare loss for couples associated with this policy.

We finally examine optimal differentiation by marital status as well as spousal labor market status (column 5). The optimal benefit levels vary substantially and wage spread increases. We have $b_0 > b_1 > b_2$ and the wage structure changes completely relative to the first three policies; we obtain $w_0 > w_2 > w_1$. Recall that an increase in b_1 is akin to an in-work subsidy and leads to wage moderation. Indeed, there is a decline in w_1 by around four percent when benefits increase from the baseline value, $b = 0.30$, to the optimal value, $b_1 = 0.48$. The insurance motive calls for generous benefits for singles ($b_0 = 0.67$) which leads to upward pressure on wages among singles. The overall welfare gain relative to the baseline amounts to almost one percent of consumption. However, couples lose whereas singles gain substantially.

Table 4. Welfare comparisons of alternative UI schemes, log utility.

	Baseline	Flat rate benefits	Constant repl. rate	Diff. by mar. status	Optimal diff.
	(1)	(2)	(3)	(4)	(5)
b	0.300	0.571			
R			0.579		
b_0			0.570	0.665	0.666
b_1			0.571	0.427	0.481
b_2			0.579	0.427	0.383
$\ln(w_1/w_0)$	0.025	0.012	0.015	0.009	−0.031
$\ln(w_2/w_0)$	0.007	0.001	0.002	−0.008	−0.005
$\ln(w_2/w_1)$	−0.019	−0.011	−0.013	−0.017	0.025
b/w_0	0.308	0.580			
b/w_1	0.301	0.573			
b/w_2	0.306	0.579			
b_0/w_0			0.579	0.672	0.673
b_2/w_1			0.579	0.428	0.399
b_1/w_2			0.579	0.435	0.489
u	0.060	0.072	0.072	0.071	0.071
t	0.019	0.043	0.043	0.041	0.043
$\Delta\Lambda^s$ (%)		1.876	1.854	3.619	3.455
$\Delta\Lambda^c$ (%)		−0.046	−0.029	−1.667	−1.501
$\Delta\Lambda$ (%)		0.915	0.913	0.976	0.977
$\Delta\Lambda^{c,s}$ (%)	2.64	0.72	0.76	−2.81	−2.40

Since singles have no access to family income sharing, they would be willing to pay something in order to gain access to a family as an insurance institution. How much would they be willing to pay? We follow the approach above and compare expected utilities for singles and couples, Λ^s and Λ^c, where employment and wage outcomes are evaluated at the utilitarian planner's solution. The difference

$\Delta\Lambda^{c,s} \equiv \Lambda^c - \Lambda^s$ is a measure of the welfare gain associated with being a member of a two-person family (ignoring non-pecuniary benefits and economies of scale) and is displayed in the last row of Table 4. For the baseline case (column 1), this gain amounts to 2.6 percent; that is, singles would be willing to pay 2.6 percent of their consumption in order to switch family status. The advantage of being a family member is almost eliminated under optimal policies of the flat rate or replacement rate varieties. Benefit differentiation by marital status as well as full optimization implies that couples would actually prefer to be singles; the welfare loss from being a member of a two-person family amounts to more than two percent of consumption.

The welfare impact of marital status is affected substantially if economies of scale are taken into account. Using a square-root scale we divided family income by $\sqrt{2}$ and found that the welfare gain associated with being member of a two-person family exceeds 30 percent (37 percent in the baseline case and 32 percent under full optimization). The optimal benefit structure, however, is independent of the square root factor. A utilitarian social planner that recognizes economies of scale would have a welfare function of the form

$$\begin{aligned}\Lambda^* \;=\;& \ln(1-t)+\gamma\,[u\ln b_0+(1-u)\ln w_0]+\\ & (1-\gamma)\left[(1-u)^2\ln\mu w_2+u^2\ln\mu b_2\right]+\\ & (1-\gamma)\left[2u(1-u)\ln\mu\left(\frac{w_1+b_1}{2}\right)\right] \qquad (31)\end{aligned}$$

where $\mu \equiv 2/\sqrt{2}$. This is equivalent to

$$\Lambda^* = \Lambda + (1-\gamma)\ln\mu \qquad (32)$$

Allowing for economies of scale does not affect optimal policies since the last term on the right-hand side of (32) is constant.

5 Concluding Remarks

We have developed a model of family job search with individual wage bargaining and examined its implications for equilibrium wage differentials and optimal unemployment insurance. Equilibrium wage differentials arise among risk-averse individuals who are *ex ante* identical. In general, there is a wage premium for workers in two-person households compared to singles when the UI system involves flat rate benefits or constant replacement rates. The wage structure responds to benefit differentiations based on marital status and spousal labor market status. The optimal UI system entails relatively high benefits for unemployed singles as well as relatively high wages for employed singles.

Several extensions of the model are conceptually straightforward. For example, it would be possible to introduce endogenous search effort, an extension that probably will predict unemployment differences between singles and couples. Our current version of the model with exogenous search effort effectively imposes identical search effort across groups, an assumption implying that unemployment rates are independent of marital status.

We have assumed income sharing in the family at 50/50, equivalent to assuming equal within-family bargaining power of the spouses. It is likely that allowing for gender differences in within-family bargaining power would lead to gender differences in labor market outcomes. As it stands, the model is silent about gender wage differentials and it cannot explain the empirical finding of a marriage premium for males but not for females.

It might also be worthwhile to consider a generalization of the model that involves endogenous determination of marital status. Benefit differentiation by marital status affect incentives for marital decisions. Some recent papers have studied the analogous problem how differential tax treatment of married and single people affect marital decisions (e.g. Chade and Ventura, 2005). Differential benefit or tax treatment of singles and couples may also increase the temptation to misreport cohabitation status.

Finally, it is noteworthy that so little of the empirical work on search and unemployment has taken the family perspective seriously into account.[10] It would be surprising if this omission were of no relevance for understanding labor market outcomes for family members.

APPENDIX 1

A Numerical Model

We assume that preferences are given by a logarithmic utility function. The time period is taken to be a quarter. The rate of interest (equal to the rate of time preference) is set to zero. Productivity is normalized to unity, $y = 1$.

A baseline case is calibrated to data for the United States. Following the estimates in Shimer (2005), we set the quarterly job destruction rate to 10 percent: $\phi = 0.10$. The matching function is Cobb Douglas, $M = au^{\eta}v^{1-\eta}$, where $\eta = 0.72$ is estimated by Shimer (2005). As Shimer we assume $\eta = \beta$. Flat rate benefits are fixed at 30 percent of productivity, $b/y = 0.3$; this implies a replacement rate

[10] The paper by Lentz (2009) is a recent exception. Lentz estimates a job search model on Danish data and finds, among other things, that spousal income matters for unemployment duration.

(relative to wages) just above 30 percent.[11] The matching parameter, a, and the vacancy cost, k, are chosen so as to obtain 6 percent unemployment, which is the average U.S. unemployment rate over the past 40 years. We also aim for a reasonably realistic relationship between the expected duration of vacancies, $1/q(\theta)$, and the expected duration of unemployment, $1/\alpha(\theta)$; the duration of vacancies is empirically much shorter than the duration of unemployment. We set $a = 2.122$ and chose a value of k that gives 6 percent unemployment: $k = 1.061$. The baseline implies that vacancy costs represent 2 percent of gross output: $kv/(1-u)y \approx 0.02$. Unemployment duration is 8.3 weeks and the duration of vacancies is 2.8 weeks. These numbers are in the same ballpark as available estimates for the United States. The estimates in Shimer (2005) imply that unemployment duration in the post-war U.S. labor market was 9 weeks on average. Information on vacancy durations is scarce, but Abraham (1983) suggests durations around one to two weeks.

The fraction of singles, γ, is set to 0.5 using information from the U.S. Census: 50 percent of the U.S. population (15+) were married with a spouse present in 2008. Table A1 presents some output implied by these parameter choices.

Table A1. A numerical model, baseline case.

θ	0.339
u	0.060
Vacancy duration (weeks)	2.8
Unemployment duration (weeks)	8.3
w_0	0.973
w_1	0.998
w_2	0.980
$\ln(w_1/w_0)$	0.025
$\ln(w_2/w_0)$	0.007
$\ln(w_2/w_1)$	-0.019
t	0.019

[11] Replacement rates in the U.S. can be well above 30 percent for insured workers; estimates hovering around 50 percent are common (e.g. OECD, 2006). But only about 35 percent of the unemployed actually receive unemployment benefits (O'Leary, 2000; Cleary et al, 2009). A representative unemployed U.S. worker faces a replacement rate well below 50 percent.

APPENDIX 2

Value Functions with Differentiated Benefits

Consider the intertemporal objective functions when benefits may differ across marital status and labor market states (wholly unemployed versus partially unemployed). For singles we have

$$\begin{aligned} rU^s &= \upsilon(\tilde{b}_0) + \alpha(N^s - U^s) \\ rN^s &= \upsilon(\tilde{w}_0) + \phi(U^s - N^s) \end{aligned}$$

and thus

$$N^s - U^s = \frac{\upsilon(\tilde{w}_0) - \upsilon\left(\tilde{b}_0\right)}{\alpha + \phi}$$

when evaluated at $r = 0$. For couples we have

$$\begin{aligned} rU^c &= \upsilon\left(\tilde{b}_2\right) + 2\alpha(E^c - U^c) \\ rE^c &= \upsilon\left(\tilde{w}_1; \tilde{b}_1\right) + \alpha(N^c - E^c) + \phi(U^c - E^c) \\ rN^c &= \upsilon(\tilde{w}_2) + 2\phi(E^c - N^c) \end{aligned}$$

The present value differences, evaluated at $r = 0$, can be written as

$$\begin{aligned} E^c - U^c &= \frac{(\alpha + 2\phi)\left[\upsilon(\tilde{w}_1; \tilde{b}_1) - \upsilon(\tilde{b}_2)\right] + \alpha\left[\upsilon(\tilde{w}_2) - \upsilon(\tilde{w}_1; \tilde{b}_1)\right]}{2(\alpha + \phi)^2} \\ N^c - E^c &= \frac{(2\alpha + \phi)\left[\upsilon(\tilde{w}_2) - \upsilon(\tilde{w}_1)\right] + \phi\left[\upsilon(\tilde{w}_1; \tilde{b}_1) - \upsilon(\tilde{b}_2)\right]}{2(\alpha + \phi)^2} \end{aligned}$$

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