Varying Life Expectancy and Social Security[1]

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Abstract

This paper studies the normative problem of redistribution between individuals who differ in their life span. We discuss important aspects related to the objective function in such a setting and argue that aversion to multi-period inequality and risk aversion with respect to the length of life should be taken into account. Then, we study the properties of the social optimum both with full information and asymmetric information.

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1 Introduction

It is well documented that life expectancy differs across socio economic groups and according to individual (random) health endowments. For example, in France, women life expectancy at age 60 is 25% larger than the one of men of the same age. Similarly, life expectancy at age 60 of a professional is 3% higher than the one of a skilled worker and 17% higher than the one of an unskilled worker. Since Social Security provides transfers contingent on survival, the combination of differential mortality and Social Security transfers may generate important redistribution, between socioeconomic groups and more generally between individuals.

Several papers measured this redistribution in financial terms (Coronado et al. (2000) and Liebman (2001)). Still there are very few normative analysis that discuss how PAYGO systems should adapt to differential mortality. Should long lived individuals work longer? Should they have lower pensions? These questions have remained undebated issues. An exception is the contribution of Diamond (2003). However, in Diamond's framework, heterogenous life expectancy only plays a role because it is assumed that there is asymmetric information on the disutiliy of work.

In this paper, we shall argue that the scarcity of studies on the design of PAYGO system with heterogenous life expectancy is due to the invisible hand that incites economists to focus on simplifying (and sometimes unappealing) assumptions. As a matter of fact, most applied papers addressing normative questions with intertemporal agents make a double assumption of additivity. First, individuals cardinal utility functions are assumed to be separable additive across time. Second the Social Welfare function is assumed to be a weighted sum of individual cardinal utility functions. This double assumption of additivity, is for example found in Brito et al. (1991), Feldstein (1987), Calvo et al. (1988), Miles et al.(2002), Golosov et al. (2003) as well as in the study of Diamond (2003) mentioned above.

The double additivity implies that the social planner preferences are separable additive with respect to individuals' instantaneous utilities. Consequently the optimal allocation (say of consumption, labor, etc...) within a given period of time is independent of the allocation in other periods of life. In particular, the optimal allocation of labor and consumption between Tim and Tom in a given period is unaffected by the fact that Tim will live much longer than Tom. The double assumption of additivity simply kills the chicken in the egg: there is not much to say about heterogenous life expectancy.

We will see, however that when this double assumption of additivity is relaxed, accounting for heterogenous life expectancy is crucial for the design of Social Security. In this paper we use a standard utilitarian approach and therefore maintains the assumption that social welfare is the sum of individual cardinal utility functions. We also keep the assumption that individuals preferences are weakly separable. The cardinal individual utility function is then obtained by a transformation of an additive utility function. The novelty is that we no longer constrain this transformation to be linear. Actually we will provide several arguments explaining why it seems reasonable to consider concave transformations. These arguments relate to aversion to multiperiod inequality or risk aversion with respect to life duration.

Our contribution does not escape from the criticism we addressed above. It is simple and relies on probably unrealistic assumptions on individual preferences (nobody really believes that the assumption of weak separability is plausible). Still it is sufficient to show that heterogeneity of life expectancy should play a key role in the design of Social Security. In our framework, the social optimum is obtained when individuals living longer retire later and consume less per period than short lived individuals.

The interest of relaxing the double additivity assumption is enhanced when looking at the problem with asymmetric information. We consider the case where agents have private information on their life expectancy and the government seeks to design an incentive compatible redistribution scheme. We illustrate this by considering two different stories. In the first one, individuals cannot save and the government commits to offer a constant annuity after retirement. Annuities are financed by a uniform and constant lump sum tax before retirement. We show that at the second best optimum individuals are provided with lower consumption after retirement than before retirement. Moreover long lived individuals retire later, with higher retirement pensions, than short lived individuals. In the second scenario, we consider the case where individuals can privately save and that savings are not observable nor at the individual nor at the aggregate level by the government. The government can only intervene through the distribution of a (positive or negative) retirement bonus that depends according to retirement age. We show that in this case, a positive or negative marginal tax rate on continued activity can be desirable depending mainly upon the cardinal measure of individuals utilities.

The paper is structured as follows. In section 2, we provide the main assumptions on individual preferences and the objective function. In section 3, we study the optimum in the context of full information and asymmetric information and section 4 concludes.

2 The Model

2.1 Individuals

There is no uncertainty in our framework. Consider an individual with life span equal to T. At each time, the individual consumes $c(t)$ and can supply either one or no unit of labor. The labor disutility at each time t is $r(t)$ with $r'(t) \geq 0$. The discount rate and the interest rates are assumed to be equal to zero. Denoting z the age of retirement, preferences are given by the following ordinal utility function:

$$V\left(c\left(t\right),z,T\right)=\int_{0}^{T}u\left(c\left(t\right)\right)dt-\int_{0}^{z}r\left(t\right)dt=\int_{0}^{T}u\left(c\left(t\right)\right)dt-R\left(z\right) \qquad (1)$$

where C is a vector of consumptions at each time and $u\left(.\right)$ is increasing and strictly concave. $R\left(z\right)=\int_{0}^{z}r\left(t\right)dt$ denotes the disutility for a working life of length z. Note that this specification assumes that preferences are separable between consumption and work. There is no heterogeneity in instantaneous labor income. The monetary unit is chosen so that the instantaneous labor income is equal to 1. At the beginning of her working life, an agent chooses among the available social security plans. Each plan is generally characterized by an age of retirement, z and a consumption path $c\left(t\right)$.

For simplicity sake we will assume that in all the cases that are considered in the paper, all individuals spend some time in retirement ($z<T$). Moreover, we will assume that for all consumption levels that are considered we have:

$$\frac{cu'(c)}{u(c)}<1 \qquad (2)$$

This latter assumption, which is only needed for the results of Section 3.2.2, is standard in economic literature that studies the welfare benefits related to longevity extension[1]. It involves assuming that the value of life is large enough.

Throughout the paper, we assume a stationary population composed of individuals differing in their life expectancy[2]. The length of life is distributed over $[T_{\min},T_{\max}]$ with a distribution function $F\left(T\right)$ and a density $f\left(T\right)$. The economy is supposed to be in a steady state equilibrium and the government can lend or borrow at the zero interest rate in order to balance the budget

[1] See for example, Murphy and Topel (2005), or Becker, Philipson and Soares (2005).

[2] This assumes implicitely that each time an individual living T years dies, a new individual with the same lifespan is born.

at any given period. The resource constraint of the economy is thus:

$$\int_{T_{\min}}^{T_{\max}} z_T f(T) dT = \int_{T_{\min}}^{T_{\max}} \left(\int_0^T c_T(t) \right) f(T) dT \tag{3}$$

where z_T and $c_T(t)$ denote the age of retirement and instantaneous consumptions of agents living T years.

2.2 The objective function

Individual ordinal utility functions being already specified, and given by $V(c, z, T)$, the natural candidates for the cardinal utility functions are of the form $G(V(c, z, T))$ where G is an increasing transformation. While the function V allows to decide whether a life is preferred to another one, the function G provides the scale for the measurement of individual happiness. Take for example two different outcomes (c_i, z_i, T_i), $i = 1, 2$ and assume that:

$$V(c_1, z_1, T_1) > V(c_2, z_2, T_2)$$

Individuals prefer life 1 to life 2. The individuals' happiness must be greater with life 1. Therefore we must have $G(V(c_1, z_1, T_1)) > G(V(c_2, z_2, T_2))$, which is fulfilled if G is increasing. However, individuals preferences are not informative on the gap between happiness with life 1 and happiness with life 2. This gap is given by

$$G(V(c_1, z_1, T_1)) - G(V(c_2, z_2, T_2))$$

and obviously depends on the function G. Knowledge of the function G is then crucial for a Social Planner that aims at maximizing the aggregate happiness in the society.

The standard assumption is that G is linear. There is no doubt that technically speaking, it is a very convenient choice. Nonetheless, there is no reason for technicality to be a relevant criterion. Below, we develop two parallel arguments to support the choice of a concave function G in our setting. The first one is related to the notion of aversion for multi-period inequality. The second one to individual preferences under uncertainty and, in particular, to risk aversion with respect to life duration.

2.2.1 Aversion for multiperiod inequality.

The concept of aversion for multiperiod inequality has been initially introduced by Atkinson and Bourguignon (1982) and more recently discussed in Gottschalk and Spolaore (2002). It can be made very intuitive by considering the simple case of a two period and two individuals setting who have preferences represented by the ordinal utility function $V(c_1, c_2) = u(c_1) + u(c_2)$. The utilitarian SWF becomes

$$SWF = G(u(c_1^1) + u(c_2^1)) + G(u(c_1^2) + u(c_2^2))$$

where c_t^i is the consumption of individual i in period t, and G, the increasing transformation that relates ordinal and cardinal utility functions. Now, consider two levels of instantaneous consumptions a and b, with $a < b$ and compare the following two social outcomes:

$$\begin{aligned} \text{Outcome A} &: \quad (c_1^1, c_2^1) = (a, a) \text{ and } (c_1^2, c_2^2) = (b, b) \\ \text{Outcome B} &: \quad (c_1^1, c_2^1) = (a, b) \text{ and } (c_1^2, c_2^2) = (b, a) \end{aligned}$$

With outcome A individual 1 has a low level of consumption in both periods while the second individual has a high level of consumption in both periods. With outcome B, both individuals alternate between low and high level of consumption. One should note that in both cases there are in each period one individual with consumption a and another one with consumption b. Thus the *within* period inequality is the same with outcomes A and B.

However, A and B differ in terms of *multiperiod* inequality. Indeed, when looking at the two periods together, outcome A seems much more unequal than outcome B: with outcome A individual 1 is worse off than individual 2, while with outcome B they have identical lifetime utility. It seems reasonable to argue that a well behaved social planner should prefer outcome B. It is straightforward to see that it is the case if and only if the function G is strictly concave. This provides our first argument for choosing a concave function G.

The concavity of the function G, and more precisely $(-G''/G')$, measures the planner's aversion for multiperiod inequality. When G is linear, $(-G''/G') = 0$ and the Social Planner is indifferent to multiperiod inequality. The greater is $(-G''/G')$ the greater is the planner's aversion for multiperiod inequality.

2.2.2 Risk aversion with respect to length of life

Another way to support the choice of a concave function G is to refer to Harsanyi's (1955) axiomatization of utilitarianism. Harsanyi considers preferences over risky alternatives and focuses on the case where the social planner and individuals have preferences that can be represented by VNM utility functions. He shows that if the Social Planner preferences satisfy an ex-ante Pareto criterion then the planner's utility function must be a linear combination of individuals utility functions. Thus, if we follow Harsanyi's axiomatic constructions, the function $G(V(c,z,T))$ has to be a VNM utility function representing individual preferences under uncertainty. Therefore, the properties of the function G can be related to fundamental characteristics of individuals' preferences. In particular, as discussed in Bommier (2006), one can make the link between G and a notion of (net) risk aversion with respect to the length of life. In the present case, where there agents have no time preferences, the link between G and risk aversion with respect to life duration is straightforward.

Consider indeed a constant path of consumption c and a retirement age z. The utility provided by living T years (with $T > z$) is $G(V(c,z,T)) =$

$G(Tu(c) - R(z))$. The absolute risk aversion with respect to the length of life, defined by $-\left(\partial^2 G(V(c,z,T))/\partial T^2\right) / \left(\partial G(V(c,z,T))/\partial T\right)$, equals then to $(-G''/G')\, u(c)$. When G is linear $-G''/G' = 0$ and individuals are risk neutral with respect to the length of life, which seems in contradiction with empirical evidence.[3] They are risk averse when G is concave, and the index of risk aversion with respect to the length of life is proportional to $-G''/G'$.

3 The social optimum

3.1 First Best Problem

Assume first that the social planner can observe the life durations. In the first best problem, a social planner chooses the consumption paths and the retirement ages in order to maximize:

$$SWF = \int G(V(c_T(t), z_T, T)) f(T) dT$$

subject to the resource constraint (3). Since the utility functions are separable, the first best solution is clearly obtained when $c_T(t) = c_T$ for every t and $T \in [T_{\min}, T_{\max}]$. The problem thus amounts to choose c_T and z_T in order to maximize

$$\int_{T_{\min}}^{T_{\max}} G\left(Tu\left(c_T\right) - R(z_T)\right) f\left(T\right) dT$$

subject to the resource constraint (3).

The first order conditions imply $r\left(z_T\right) = u'\left(c_T\right)$ and $u'\left(c_T\right) G'\left(U\left(c_T, z_T, T\right)\right)$ is independent of T. This leads to the two following propositions:

Proposition 1 *When G is linear, the first best optimum is such that for any T and $\tilde{T}$,*

(i) $c_T = c_{\tilde{T}}$

(ii) $z_T = z_{\tilde{T}}$

[3] See Bommier and Villeneuve (2005).

It is clear that such a social security scheme redistributes lifetime income from the short lived to the long lived individuals. The net contribution of a type T individual to the system is equal to $z - Tc$ where, because of the resource constraint, $z = c\int_{T_{\min}}^{T_{\max}} Tf(T)dT$. Therefore every individuals with a life duration T that is lower than the mean of the life duration $\overline{T} = \int_{T_{\min}}^{T_{\max}} Tf(T)dT$ are net contributors to the system while those who live longer are net recipients.

Proposition 2 *When G is strictly concave, the first best optimum is such that for any $T < \tilde{T}$,*

(*i*) $c_T > c_{\tilde{T}}$

(*ii*) $z_T < z_{\tilde{T}}$

At the optimum, those who live longer retire later and consume less. As opposed to the double additive approach, long lived agents may not be net recipients of the redistribution system. Whether long lived agents are net recipients or not now clearly depends upon the concavity of G. The level of aversion to multiperiod inequality is thus crucial to determine the optimal level of redistribution between individuals of different life-span.

3.2 Asymmetric information

The first best solution has been derived under the assumption that individual life spans T are observable. In the following we assume that the government cannot observe T while he can observe the retirement age and the distribution of individuals over $[T_{\min}, T_{\max}]$. In a first part, we investigate the solution of the social planner when individuals have no access to private savings while the government provides individuals with constant retirement pensions. Then we turn to the case where individuals can privately save.

3.2.1 Asymmetric information without private savings

Without any further constraint, the first best optimum described in propositions 1 and 2 is still implementable when individuals cannot privately save. If the government proposes first best bundles (c_T, z_T) where c_T is offered until age T and 0 after, long lived individuals would not have any interest to mimic short lived individuals.

Suppose instead that the government commits to offer a constant consumption path (or annuity) c_T^2 after retirement without condition on age. In this case, the first best allocations are not incentive compatible. Individuals would optimally choose to claim to be short lived so as to enjoy a higher per period consumption (or annuity) and a shorter career (except when G is linear since it is a pooling optimum). Under these assumptions, the problem of the government is thus the first best problem to which we add a global incentive constraint preventing any type of individuals to mimic the other types of individuals. In order to avoid technical computational difficulties, we make the additional assumption that pre retirement consumptions denoted c^1 should be uniform across types and not age dependent. The system can thus be interpreted as a retirement system with uniform contributions and a menu of retirement age z_T and retirement benefits c_T^2. This models a world where individual only reveal their type at retirement. Formally, the government's problem becomes:

$$\max_{c^1 c_T^2,\, z_T} \int_{T_{\min}}^{T_{\max}} G\left(z_T u\left(c^1\right) + (T - z_T)\, u\left(c_T^2\right) - R(z_T)\right) f(T)\, dT$$

$$\int_{T_{\min}}^{T_{\max}} \left(z_T - z_T c^1 - (T - z_T)\, c_T^2\right) f(T)\, dT \ \geq 0$$

$$z_T u\left(c^1\right) + (T - z_T)\, u\left(c_T^2\right) - R(z_T) \geq z_{T'} u\left(c^1\right) + (T - z_{T'})\, u\left(c_{T'}^2\right) - R(z_{T'}) \qquad \forall T, T'$$

The complete solution of this problem is given in the appendix. Below, we just sketch the results that are crucial for our analysis. We determine wether the consumption paths and the retirement ages follow the same properties as in proposition 2.

Using the definition of $V\left(c^1, c^2_T, z_{T,}T\right)$, the incentive compatibility constraints can be rewritten as

$$V\left(c^1, c^2_T, z_{T,}T\right) \geq V\left(c^1, c^2_{T'}, z_{T',}T'\right) + \left(T - T'\right) u\left(c^2_{T'}\right) \qquad \forall T, T' \tag{4}$$

This requires that for T' approaching T, we have in the limit $\dot{V}_T = u\left(c^2_T\right)$ for all T where a dot means that the variable is derived with respect to T. Moreover given that $u\left(c^2_T\right)$ is the slope of V_T at T, the inequality in (4) requires that V_T be convex. This is fulfilled if and only if $\dot{c}^2_T \geq 0$. In other words, the optimum always involves a non decreasing post retirement consumption path with respect to the life duration. In the appendix, we show the following:

Proposition 3 *For any $T, \tilde{T} \in [T_{\min}, T_{\max}]$ such that $T < \tilde{T}$, the second best optimum is characterized by:*

(i) $c^2_T \leq c^2_{\tilde{T}}$

(ii) $z_T \leq z_{\tilde{T}}$

(iii) $c^1 > c^2_T$ *for every* $T \in [T_{\min}, T_{\max}[$ *and* $c^1 = c^2_{T_{\max}}$.

To understand point *(iii)*, note that the type T marginal rate of substitution between c^1 and c^2 writes as follows:

$$MRS^T_{c^1,c^2} = -\frac{(T - z)\, u'\left(c^2\right)}{z u'\left(c^1\right)}$$

which decreases with T for given levels of c^1, c^2 and z. In other words, for a given level of c_1 and z, short lived individuals must be compensated less after retirement to accept a given level of reduction of their before retirement consumption. Thus distorting downward the choice of c^2 is a way to relax an otherwise binding self selection constraint. For individuals at the top of the distribution, the usual "no distortion at the top" result holds so that (in the limit) $c^1_{T_{\max}} = c^2_{T_{\max}}$. We also show in the appendix that there is always bunching at the bottom. In other words, on some given interval $[T_{\min}, T]$, individuals are offered the same allocation.

3.2.2 Asymmetric information with private savings

Assume now that individuals can privately save. We assume further that private accumulated wealth is not observable nor at the individual nor at the aggregate level so that savings cannot be taxed either linearly or non linearly. The government thus only observes the retirement age and gets only one redistributive tool to improve social welfare: he can distribute a (positive or negative) "retirement bonus" depending on retirement age. First best contracts as described in propositions 1 and 2 are generally not implementable.

Denote by $B(z)$ the retirement bonus. Denote by I the cycle income (including the retirement bonus) given by:

$$I = z + B(z)$$

With these notations, the retirement age chosen by a type T individual is given by:

$$MRS^T_{Iz} = \frac{r(z_T)}{u'(c_T)} = 1 + B'(z_T) \tag{5}$$

where MRS_{Iz} stand for the marginal rate of substitution between life cycle net income and the retirement age. Clearly, if $B'(z_T) > 0$ (resp. < 0) there is a subsidy (resp. tax) on prolonged activity. For further use, denote by $\omega = 1 + B'(z_T)$ the life cycle net productivity and ϵ the elasticity of the net life cycle income I with respect to ω for a given level of z. Differentiation of (5) gives:[4]

[4] The relationship between ϵ and the compensated elasticity of labor supply ϵ_{zz} is the following: $\epsilon_{zz} = \frac{r(z)}{\left[r'(z)+\frac{\omega^2}{T}u''(c_T)\right]}$ or $\frac{1}{\epsilon_{zz}} = \frac{zr'(z)}{r(z)} + \frac{1}{\epsilon}$..

$$\epsilon(T) = \frac{dI}{d\omega}\frac{\omega}{I} = \frac{u'(c_T)}{u''(c_T)\,c_T} < 0$$

The program of the government is to choose chooses contracts (I_T, z_T) by solving:

$$\max_{c_T,\, z_T} \int_{T_{\min}}^{T_{\max}} G\left(Tu\left(\frac{I_T}{T}\right) - R(z_T)\right) f(T)\, dT$$

$$\int_{T_{\min}}^{T_{\max}} (z_T - I_T)\, f(T)\, dT \geq 0$$

$$T\left(\frac{I_T}{T}\right) + R(z_T) \geq Tu\left(\frac{I_{T'}}{T}\right) - R(z_{T'}) \qquad \forall T, T'$$

Each individual of type T chooses $(I_{T'}, z_{T'})$ such that it solves

$$\max_{T'} Tu\left(\frac{I_{T'}}{T}\right) - R(z_{T'})$$

This yields the following first order local condition:

$$\dot{V}_T = u\left(\frac{I_T}{T}\right) - \frac{I_T}{T}u'\left(\frac{I_T}{T}\right) \tag{6}$$

where a dot means that the variable is derived with respect to T and the second order local condition is $\dot{I}_T$ and $\dot{z}_T \geq 0$.[5] To interpret this last equation, one may rewrite it as:

$$\dot{V}_T = u\left(\frac{I_T}{T}\right)\left[1 - \frac{\frac{I_T}{T}u'\left(\frac{I_T}{T}\right)}{u\left(\frac{I_T}{T}\right)}\right]$$

which is positive by assumption (see equation (2)).

In the appendix we show that the optimum can be described by the following equation:

$$\frac{B'(z_T)}{1 + B'(z_T)} = -u'(c_T)\frac{\lambda(T)}{Tf(T)}\frac{1}{\epsilon(T)} \tag{7}$$

[5] From now we assume that the first order approach is valid (i.e. that $\dot{I}_T > 0$ and $\dot{z}_T > 0$).

where

$$\lambda(T) = \int_{T}^{T^{\max}} \left[\frac{1}{r(z_x)} - \frac{G'(V_x)}{\gamma}\right] f(x)\, dx \tag{8}$$

with $\lambda(T_{\min}) = \lambda(T_{\max}) = 0$.

The expression for the marginal tax in (7) has a very similar form to the one obtained in non linear income tax model (e.g. see Atkinson and Stiglitz (1980)).[6] Increasing the marginal tax rate has distributional consequences measured by $\lambda(T)$. This term measures the social net gain associated with a marginal decrease of the utility for every individuals who live more than T years. The gain in increased revenue is $1/r(z_x)$ per person while the cost is a loss of welfare measured in units of revenue by $G'(V_x)/\gamma$. It also has a distortionary effect with magnitude measured by $\epsilon(T)$: the higher is $\epsilon(T)$ the higher is the distortion so the lower will be the marginal tax rate. Finally $Tf(T)$ measures the mass of type T individuals.

It turns out that the sign of the marginal tax rate thus only depends upon the sign of $\lambda(T)$. In the appendix, we show the following:

Proposition 4 *(i)If G is linear then $B'(z_T) > 0$ for $T \in]T_{\min}, T_{\max}[$.*

(ii) If G is strictly concave and R is linear, then $B'(z_T) < 0$ for $T \in]T_{\min}, T_{\max}[$.

(iii) If G is Rawlsian then $B'(z_T) < 0$ for $T \in [T_{\min}, T_{\max}[$.

To understand this proposition, first note that differentiation of $MRS_{Iz}^{T}(I,z) = r(z)/u'(I/T)$ with respect to T yields:

$$\frac{dMRS_{Iz}^{T}(I,z)}{dT} = MRS_{Iz}^{T}(I,z)\left[\frac{c_T u''(c_T)}{u'(c_T)}\right] < 0$$

so that long lived individuals must be compensated less to accept an increase in their length of activity. This implies that if the government wishes to redistribute life cycle income from the short lived towards the long lived,

[6] In these models, the parameter of heterogeneity is labor productivity.

an upward distortion on the retirement age is a way to relax incentive constraints. This makes short lived individuals be less attracted by bundles offered to long lived individuals. Following the reverse reasoning, if the government wishes to redistribute towards the short lived, then taxing the continued activity is a way to relax incentive constraints. Thus the sign of the distortion imposed on the retirement age comes from the direction of redistribution desired by the government which is reflected in $\lambda(T)$.

When G is linear, the government wishes to redistribute life cycle income towards those who live long. This is reflected in (8) where following a decrease in utility, the gain in increased revenue $1/r(z_x)$ is decreasing with the life expectancy (remember that $\dot{z} > 0$) while the welfare cost is constant. In this case, $\lambda(T)$ is increasing from 0 up to $\lambda(T_k)$ where $1/r(z_{T_k}) = G'(V_{T_k})/\gamma$ and then decreases to 0. This makes the government willing to redistribute from the short to the long lived. As argued above, this pleads in favor of a marginal subsidy on continued activity.

However, when $R(.)$ is linear and $G(.)$ is strictly concave, the gain in increased revenue is constant and the welfare loss is decreasing with the life expectancy (since $\dot{V}_T > 0$). In this case, $\lambda(T)$ is always negative. By the same way, when $G(.)$ is Rawlsian, the loss in welfare is only positive for the individual with the smallest life expectancy so that $\lambda(T) < 0$ and a tax on continued activity is desirable in order to ease redistribution from the long lived towards the short lived.

4 Conclusion

Maximization of a Social Welfare function subject to resource and incentive constraints often proves to be a difficult and technical task. In practice, economists resort to simplifying assumptions in order to maintain the difficulty at reasonable level. It is for no other reason that normative issues concerning intertemporal agents are generally addressed under the assumption that individual cardinal utility functions are separable additive.

Ethically speaking, such an assumption is hardly defendable. Each individual life is modelled as a sequence of independent incarnations and the notions of justice between individuals is replaced by a notion of justice between incarnations. It is assumed that there is no more connection between Tim in year 0 and Tim in year 1 than between Tim in year 0 and Tom in year 1. It is the nature of human existence that is simply denigrated.

Simplifying assumptions may lead to ignore important questions. One of them is the question of the optimal design of Social Security with heterogeneity in life expectancy. As long as individuals' cardinal utilities are assumed to be additively separable there is little to say on that topic. In such a case (with individuals that only differ by their life expectancy) the first best optimum is then simply obtained by setting consumption and retirement independently of life expectancy.

Differential mortality proves however to play a central role, when the assumption of additive cardinal utilities is relaxed. A Social Planner who exhibits aversion to multiperiod inequality prefers long lived individual to work longer and have lower instantaneous consumptions than short lived individuals. In other words, labor and consumption should be allocated according to another factor that crucially impacts individual's welfare: life duration.

This new perspective gives birth to a whole set of public economics problems, since life expectancy is closely related to individual's health, which for a number of technical and ethical reasons, is an asymmetric information. As an illustration, we have considered two extreme cases, that can be seen as very stylized representation Pay-as-You-Go and Capitalization retirement systems. In the first one individual have no access to the credit market and exclusively rely on pensions during their retirement. In the second there is a perfect credit market, and the only public policy involves lump-sum transfers conditional on retirement age. In both cases, we found that accounting for the Planner's aversion to multiperiod inequality impacts, both quantitatively and qualitatively, the impact of differential mortality, bringing new insights on the optimal replacement rate in a Pay-as-You go pension system or on the retirement transfers in a capitalization one.

In order to emphasize the central role of cardinal assumptions, and in particular of additivity, we focused on a very simplified model, where key elements such as uncertainty about the timing of death where ignored. Obviously these elements should be reintegrated in the analysis in order to provide more realistic model. This is on our research agenda.

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Appendix

A The second best optimum without private savings

The second best problem can be transformed into a standard optimal control problem. Using the local incentive constraints $\dot{V}_T = u\left(c_T^2\right)$ and $\dot{c}_T^2 \geq 0$ where a dot means that the variable is derived with respect to T, the problem can be rewritten as :

$$\begin{array}{l}
\underset{c^1, c_T^2,\, z_T}{Max} \quad \int_{T_{\min}}^{T_{\max}} G\left(V_T\right) f\left(T\right) dT \\
s.to: \ \int_{T_{\min}}^{T_{\max}} \left\{z_T - z_T c^1 - \left(T - z_T\right) c_T^2\right\} f\left(T\right) dT \ \geq 0, \\
V_T = z_T u\left(c^1\right) + \left(T - z_T\right) u\left(c_T^2\right) - R(z_T), \\
\dot{V}_T = u\left(c_T^2\right), \\
\dot{c}_T^2 \geq 0
\end{array}$$

The state variables are V_T, c_T^2 and control variables are y_T, c_T^1 and z_T where $y_T \equiv \dot{c}_T^2$. The Hamiltonian is:

$$\begin{aligned}
H \;=\;& G\left(V_T\right) f\left(T\right) - \alpha\left(T\right)\left(V_T - z_T u\left(c^1\right) - \left(T - z_T\right) u\left(c_T^2\right) + R(z_T)\right) \\
&+ \gamma\left\{z_T - z_T c^1 - \left(T - z_T\right) c_T^2\right\} f\left(T\right) + \lambda\left(T\right) u\left(c_T^2\right) + \mu\left(T\right) y_T + \pi\left(T\right) y_T
\end{aligned}$$

where $\lambda\left(T\right)$ is the co-state variable associated with $\dot{V}_T = u\left(c_T^2\right)$, $\mu\left(T\right)$ is the co-state variable associated with $\dot{c}_T^2 \equiv y_T$. $\alpha\left(T\right)$ is the shadow value of the constraint $V_T = z_T u\left(c^1\right) + \left(T - z_T\right) u\left(c_T^2\right) - R(z_T)$, $\pi\left(T\right)$ is the shadow value of the constraint $\dot{c}_T^2 \geq 0$ and γ is the Lagrange multiplier associated with the resource constraint.

From the Pontryagin principle,

$$\dot{\lambda}\left(T\right) = -\frac{\partial H}{\partial V_T\left(T\right)} = \alpha\left(T\right) - G'\left(V_T\right) f\left(T\right) \tag{9}$$

$$\dot{\mu}\left(T\right) = -\frac{\partial H}{\partial c_T^2} = -\left[(T - z_T)\alpha\left(T\right) + \lambda\left(T\right)\right] u'\left(c_T^2\right) + (T - z_T)\gamma f\left(T\right) \tag{10}$$

Optimizing with respect to z_T, c^1 and y_T also yields:

$$\frac{\partial H}{\partial y_T} = \mu(T) + \pi(T) = 0 \tag{11}$$

$$\frac{\partial H}{\partial z_T} = -\alpha(T)\left[r(z_T) + u\left(c_T^2\right) - u\left(c_T^1\right)\right] + \gamma\left(1 - c_T^1 + c_T^2\right) f(T) = 0 \tag{12}$$

$$\frac{\partial H}{\partial c^1} = \alpha(T) z_T u'\left(c^1\right) - \gamma z_T f(T) = 0 \tag{13}$$

The transversality conditions are:

$$\lambda(T_{\min}) = \lambda(T_{\max}) = \mu(T_{\min}) = \mu(T_{\max}) = 0$$

$$\pi(T) > 0 \rightarrow \dot{c}^2(T) = 0; \quad \pi(T) = 0 \rightarrow \dot{c}^2(T) > 0 \tag{14}$$

Lemma 1 *If $\dot{c}_T^2 > 0$ then $c_T^1 > c_T^2$ for $T \in]T_{\min}, T_{\max}[$ and $c_{T_{\min}}^1 = c_{T_{\min}}^2$ and $c_{T_{\max}}^1 = c_{T_{\max}}^2$.*

Proof. Suppose that $\dot{c}_T^2 > 0$. Using (13) and (10), one has:

$$\frac{u'\left(c^1\right)}{u'\left(c_T^2\right)} = 1 + \frac{\lambda(T)}{\alpha(T)(T - z_T)} \tag{15}$$

Using (9) and the transversality conditions, we have:

$$\lambda(T) = \int_T^{T^{\max}} \left[G'(V_x(x)) f(x) - \alpha(x)\right] dx$$

Using (13) to substitute for $\alpha(T)$, this gives:

$$\lambda(T) = \int_T^{T^{\max}} \left[G'(V_x(x)) - \frac{\gamma}{u'(c^1)}\right] f(x)\, dx \tag{16}$$

$\lambda(T)$ measures the social net gain associated with a marginal decrease of the utility of every individuals who live more than T years. The gain in increased revenue is $\gamma/u'\left(c^1\right)$ per person while the cost is a loss of welfare measured by $G'(V_x(x))$. $= G'(V_{T_k}(T_k))/\gamma$. Because the utility is increasing with T (from local incentive constraints, $\dot{V}_T(T) = u\left(c_T^2\right)$) and G is concave (the social planner puts more preference towards those who have a short life), the integral in (16) is decreasing from 0 up to T_k where $G'(V_{T_k}) = \gamma/u'\left(c^1\right)$ and then increasing up to 0.Since $\lambda(T) < 0$ for $T \in]T_{\min}, T_{\max}[$and $\alpha(T) > 0$, one gets $u'\left(c_T^1\right) < u'\left(c_T^2\right)$ by (15) so that by concavity of $u(.)$, $c_T^1 > c_T^2$. By the transversality conditions (14), $c_{T_{\min}}^1 = c_{T_{\min}}^2$ and $c_{T_{\max}}^1 = c_{T_{\max}}^2$. ■

Lemma 2 *If* $\dot{c}_T^2 > 0$ *then* $\dot{z}_T > 0$.

Proof. Assume that $\dot{c}_T^2 > 0$ so that $\pi(T) = \mu(T) = 0$. In response to functions (c^1, c_T^2, z_T), individuals with type T maximize their utility:

$$\underset{T'}{Max} V\left(c_{T'}^1, c_{T'}^2, z_{T'}, T\right) = z_{T'} u\left(c^1\right) + (T - z_{T'})\, u\left(c_{T'}^2\right) - R(z_{T'})$$

The first-order condition is:

$$\dot{z}_{T'} u\left(c_{T'}^1\right) + (T - z_{T'})\, u'\left(c_{T'}^2\right) \dot{c}_{T'}^2 - \dot{z}_{T'} u\left(c_{T'}^2\right) - r(z_{T'})\, \dot{z}_{T'} = 0$$

Since revealing his true type is an optimal strategy this yields:

$$\dot{z}_T \left[u\left(c^1\right) - u\left(c_T^2\right) - r(z_T)\right] + (T - z_T)\, u'\left(c_T^2\right) \dot{c}_T^2 = 0$$

From the expression above ,we get:

$$\dot{z}_T = -\frac{(T - z_T)\, u'\left(c_T^2\right) \dot{c}_T^2}{u\left(c_T^1\right) - u\left(c_T^2\right) - r(z_T)} \tag{17}$$

where $u\left(c_T^1\right) - u\left(c_T^2\right) - r(z_T)$ is the disutility of labor which is negative so that $\dot{z}_T > 0$. ■

Lemma 3 $\dot{c}_{T_{\min}}^2 = 0$ *and* $c^1 > c_{T_{\min}}^2$.

Proof. Suppose that $\dot{c}_T^2 > 0$ at $T = T_{\min}$. Then by (15) and the transversality condition, one would have $c^1 = c_{T_{\min}}^2$. But since $\dot{c}_T^2 > 0$ for some $T > T_{\min}$ and $c^1 = c_{T_{\max}}^2$, this is impossible. Moreover, since $\dot{c}_T^2 \geq 0$ for every $T > T_{\min}$ and $c^1 = c_{T_{\max}}^2$, one necessarily has $c^1 > c_{T_{\min}}^2$. ■

A.1 Proof of proposition 3

The proof is obtained by a combination of lemma 1, 2 and 3.

B The second best optimum with private savings

The second best problem can be transformed into a standard optimal control problem. Using the local incentive constraints $\dot{V}_T = u\left(c_T^2\right)$ and $\dot{c}_T^2 \geq 0$, the problem can be rewritten as:

$$
\begin{aligned}
&\underset{I_T,\, z_T}{Max}\ \int_{T_{\min}}^{T_{\max}} G\left(V_T\right) f\left(T\right) dT \\
&s.to:\ \int_{T_{\min}}^{T_{\max}} \left\{z_T - I_T\right\} f\left(T\right) dT\ \geq 0, \\
&V_T = Tu\left(\tfrac{I_T}{T}\right) - R(z_T), \\
&\dot{V}_T = u\left(\tfrac{I_T}{T}\right) - \tfrac{I_T}{T}u'\left(\tfrac{I_T}{T}\right), \\
&\dot{I}_T, \dot{z}_T \geq 0
\end{aligned}
$$

Assuming the first approach is valid we have a state variable V_T and control variables are I_T and z_T. The Hamiltonian is:

$$
\begin{aligned}
H &= G\left(V_T\right) f\left(T\right) - \alpha\left(T\right)\left(V_T - Tu\left(\frac{I_T}{T}\right) + R(z_T)\right) \\
&\quad + \gamma\left\{z_T - I_T\right\} f\left(T\right) + \lambda\left(T\right)\left[u\left(\frac{I_T}{T}\right) - \frac{I_T}{T}u'\left(\frac{I_T}{T}\right)\right] + \mu\left(T\right) y_T + \pi\left(T\right) y_T
\end{aligned}
$$

where $\lambda\left(T\right)$ is the co-state variable associated with $\dot{V}_T = u\left(\frac{I_T}{T}\right) - \frac{I_T}{T}u'\left(\frac{I_T}{T}\right)$, $\alpha\left(T\right)$ is the shadow value of the constraint $V_T = Tu\left(\frac{I_T}{T}\right) - R(z_T)$, and γ is the Lagrange multiplier associated with the resource constraint. From the Pontryagin principle,

$$
\dot{\lambda}\left(T\right) = -\frac{\partial H}{\partial V_T} = \alpha\left(T\right) - G'\left(V_T\right) f\left(T\right) \tag{18}
$$

Optimizing with respect to I_T and z_T also yields:

$$
\frac{\partial H}{\partial I_T} = \alpha\left(T\right) u'\left(\frac{I_T}{T}\right) - \lambda\left(T\right)\frac{I_T}{T^2}u''\left(\frac{I_T}{T}\right) - \gamma f\left(T\right) = 0 \tag{19}
$$

$$
\frac{\partial H}{\partial z_T} = -\alpha\left(T\right) r\left(z_T\right) + \gamma f\left(T\right) = 0 \tag{20}
$$

The transversality conditions are:

$$
\lambda\left(T_{\min}\right) = \lambda\left(T_{\max}\right) = 0 \tag{21}
$$

Substituting (20) in (18) and integrating between T and $T_{\max}$ yields:

$$
\begin{aligned}
\lambda\left(T\right) &= \int_T^{T^{\max}} \left[\alpha\left(x\right) - G'(V_x\left(x\right)) f\left(x\right)\right] dx \\
&= \gamma \int_T^{T^{\max}} \left[\frac{1}{r\left(z_x\right)} - \frac{G'(V_x\left(x\right))}{\gamma}\right] f\left(x\right) dx \qquad (22)
\end{aligned}
$$

where we used (21.). Moreover, combining equations (19) and (20) yield:

$$MRS_{Iz}^{T}=1-\frac{\lambda\left(T\right)}{\alpha\left(T\right)}\frac{I_{T}}{T^{2}}\frac{u''\left(\frac{I_{T}}{T}\right)}{u'\left(\frac{I_{T}}{T}\right)} \tag{23}$$

Lemma 4 *If G is linear then $\lambda\left(T\right)>0$ for $T\in\left]T_{\min},T_{\max}\right[$.*

Proof. If G is linear then $G'(V_{x}\left(x\right))$ is a constant. Since $\lambda\left(T_{\min}\right)=\lambda\left(T_{\max}\right)=0$, the integral in (22) is increasing from 0 up to T_{k} where $G'\left(V_{T_{k}}\left(x\right)\right)/\gamma=1/r\left(z_{T_{k}}\right)$ and then decreasing down to 0. ■

Lemma 5 *If G is strictly concave and $R\left(z\right)$ is linear, then $\lambda\left(T\right)<0$ for $T\in\left]T_{\min},T_{\max}\right[$.*

Proof. If $R\left(z\right)$ is linear, then $1/r\left(z_{x}\right)$ is a constant. Since $\lambda\left(T_{\min}\right)=\lambda\left(T_{\max}\right)=0$, the integral in (22) is decreasing from 0 up to T_{k} where $G'\left(V_{T_{k}}\left(x\right)\right)/\gamma=1/r\left(z_{T_{k}}\right)$ and then increasing up to 0. ■

Lemma 6 *If G is of the maximin form then $\lambda\left(T\right)<0$ for $T\in\left[T_{\min},T_{\max}\right[$.*

Proof. When G is maximin, one can rewrite $\lambda\left(T\right)$ as

$$\lambda\left(T\right)=1-\int_{T_{\min}}^{T}\alpha\left(x\right)dx$$

where $\int_{T_{\min}}^{T_{\max}}\alpha\left(x\right)dx=1$ and $\alpha\left(x\right)>0$ for every x by (20) so that $\lambda\left(T\right)<0$ for every $T\in\left[T_{\min,}T_{\max}\right[$ and $\lambda\left(T_{\max}\right)=0$. ■

B.1 Proof of proposition 4

The proof is obtained by combination of lemma 4, 5 and 6 and equation (23).